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Modelling the micro-hydraulic system in MyHand2 by Hy5

Credit: Hy5.no

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Abstract

This project aims to create a mathematical model of the pressure dynamics in the hydraulic system of $MyHand2^{TM}$, a hand prosthetic made by the company Hy5. The purpose is to use the model for three things: better understanding of how hydraulic parameters affect the pressure dynamics, improve the design of the hydraulic system in order to reduce the pressure losses, and use the model for automatic control of the fingers movements in order to improve gripping and the user experience.

$MyHand2^{TM}$ have problems with unexpected pressure drops and noise in the hydraulic system. A better understanding of the hydraulic system is important for addressing these issues. Both issues have been investigated previously, with an attempt at modeling and simulating the system in COMSOL. This proved difficult as the model became too complex to simulate. Therefore, this project tries a different approach by designing a purely mathematical dynamic model which can be simulated in MatLab.

The method of this project starts with a literature study to find out what kind of hydraulic modelling that has been done before and look for sources which can be used for this project. For modelling the hydraulic system, physical balance laws are employed on the system, using the methods found in the literature study as a resource. The model is then simulated using MatLab with the ODE solver ode45.

The model resulted in a simplified system using several assumptions and simplifications: zero fluid compression in flow equations, no impulse delay for pressure propagation, no pressure loss in transmission pipes, static pressure in pump, linear spring force, no valves and no stiction.

The simulations showed that this model's general behavior makes sense in terms of the real system. However, no measurement data exists from the real hand, so it is unknown how the simulation reflects the real hand in terms of accuracy and details. Tuning, questioning of assumptions, and further development of the model will need to be done through validation against measured pressure data.

This project serves as a preparatory project for a master thesis, which will focus on taking measurements from $MyHand2^{TM}$, validating the model, make appropriate adjustments to the model, and then suggest hydraulic design improvements for $MyHand2^{TM}$.

Chapter 1

Introduction

This project aims to create a mathematical model of the pressure dynamics in the hydraulic system of $MyHand2^{TM}$, a hand prosthetic made by the company Hy5. The purpose is to use the model for three things: better understanding of how hydraulic parameters affect the pressure dynamics, improve the design of the hydraulic system in order to reduce the pressure losses, and use the model for automatic control of the fingers movements in order to improve gripping. The project is a preparatory project for a master thesis, which will focus on taking measurements from $MyHand2^{TM}$, validating the model, make appropriate adjustments to the model, and then suggest hydraulic design improvements for $MyHand2^{TM}$.

Hy5 have already launched the first version of the hand, but there have been two problems with the system that could be improved. First, the valves that controls the movement of the fingers has been shown to experience large pressure drops when running the system. And second, the system is producing unwanted noise which is harming the user experience. This noise could be a result of the pressure drops, causing cavitation within the hydralic system. It is important to solve these issues, and it is believed that having a mathematical model of the system can help in understanding the causes of the pressure drops.

Previous research addressing these issues has focused on two methods. The first method were based on recording the sound noise and analyzing the frequency spectrum, with the findings that the noise frequency seems to correlate with the frequency of the gears in the pump. The second method focused on modeling the pump and valves from MyHand with the software COMSOL. The modeling and simulation with COMSOL were unsuccessful, and it were concluded that COMSOL probably is not a good software for this type of simulation (Engmark (2019)).

This report takes a different approach by deriving a mathematical dynamic model of the hydraulic system through physical balance laws and mathematical principles. The model is then simulated in MatLab with an ODE solver ode45.

1.1 Problem statement

The company Hy5 is creating the world first hand prosthesis based on micro-hydraulics.

The valves controlling the movement of the fingers has shown to have large pressure drops, and it is experienced noticable sound noice when the hand is running. Therefore, it is desired to gain a better understanding of the hydraulic system, in terms of the pressure dynimcs, by creating a mathematical model of the system. The model will focus on pressure dynamics, pressure drops and flow through the hydralic valves.

The hope is that the model can be used for gaining a better understanding of the hydraulic system, improving the design of the hydralic system, and use the model for controlling the pressure through closed loop control.

Tasks:

1. Literature study
2. Model the hydraulic system in MyHand 2, with focus on pressure drops and flows
3. Implement the model in MATLAB
4. Explore through simulation how different modelling parameters affect pressure drops and flows

Supervisors:

- Professor Ole Morten Aamo, Department of Engineering Cybernetics, NTNU
- Bjørn Olav Bakka, CTO Hy5 AS

1.2 Description of MyHand 2

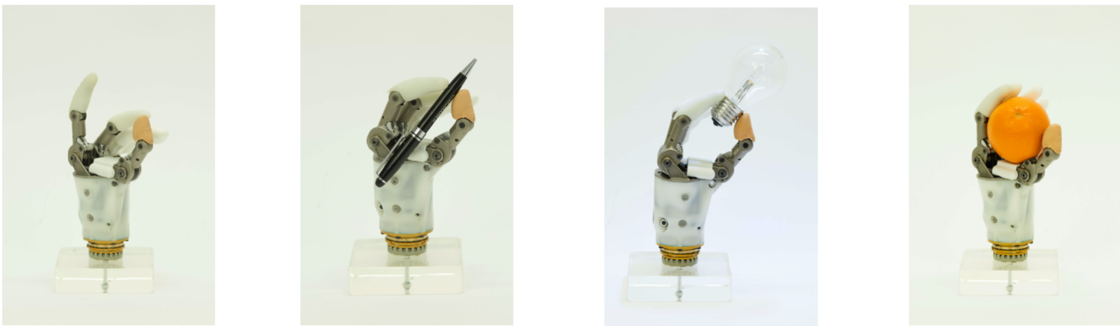

Figure 1.1: Image showing different grip types of $MyHand^{TM}$. *Credit: Hy5 (2018c).*

$MyHand^{TM}$ is a prosthetic hand which uses micro-hydraulics for driving the movement of the fingers. Different grip types are possible (fig. 1.1). The gripping is controlled by myoelectric signals from the user's forearm muscles, which gives a voltage signal to a motor through a microcontroller. The motor drives a hydraulic gear pump, which provides the flow to the actuators of the fingers. Three of the fingers have actuators, while the remaining two fingers (ring and little finger) follows the movement of the other fingers.

Several valves exists within the hand. A security valve which opens if the pressure becomes too high. Solenoid valves for the actuators, stopping the fluid to flow backwards until given a signal. For the thumb an additional solenoid valve stops fluid in the direction of the actuator so that the microcontroller can choose when to activate the thumb.

The thumb has a position sensor, which measures the position of the thumb and providing this information to the controller. The controller can therefore monitor the position and determine when the thumb is at the appropriate position. There are also a pressure sensor on the high pressure side (the side of the hydraulic system where the pressure is increasing during gripping), between the actuators for the index and long finger, which the controller monitors as well. Through the sensors and solenoid valves, the controller can provide several control options for the hand.

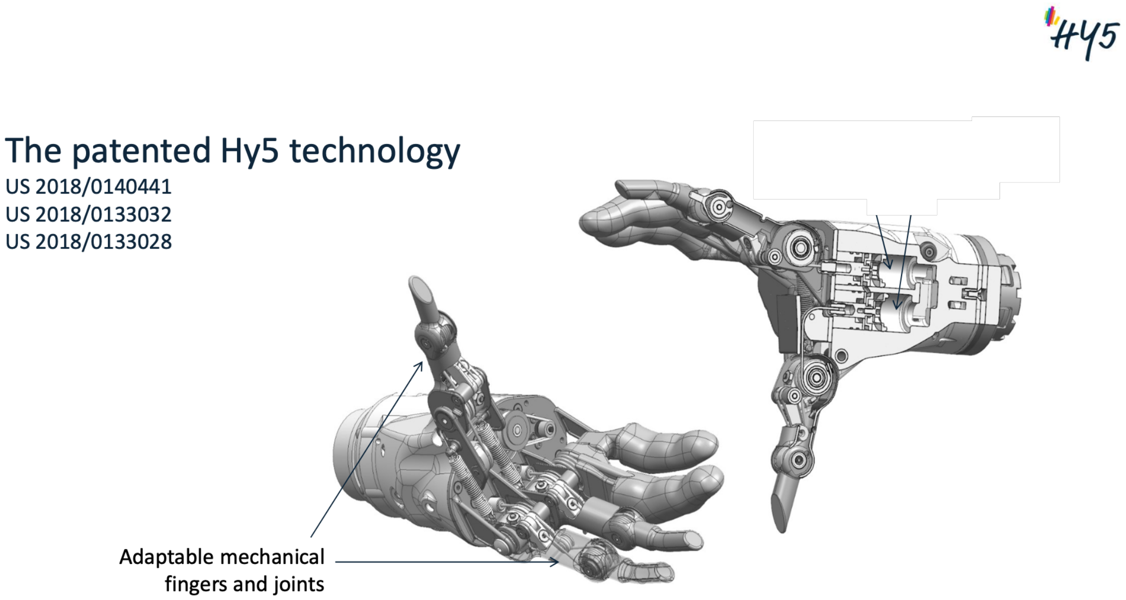

Figure 1.2: Image showing a detailed CAD drawing of $MyHand^{TM}$. *Credit: Hy5 (2018c).*

The pistons in the actuators are connected to a wire which pulls on the fingers. The fingers and the pistons driving the fingers have springs attached to them, pushing the fingers towards a natural opened state. The pump has to provide enough pressure to drive the pistons against these spring forces in order to close the hand. In figure 1.2 we can see a CAD drawing of the fingers, the springs, and the actuators.

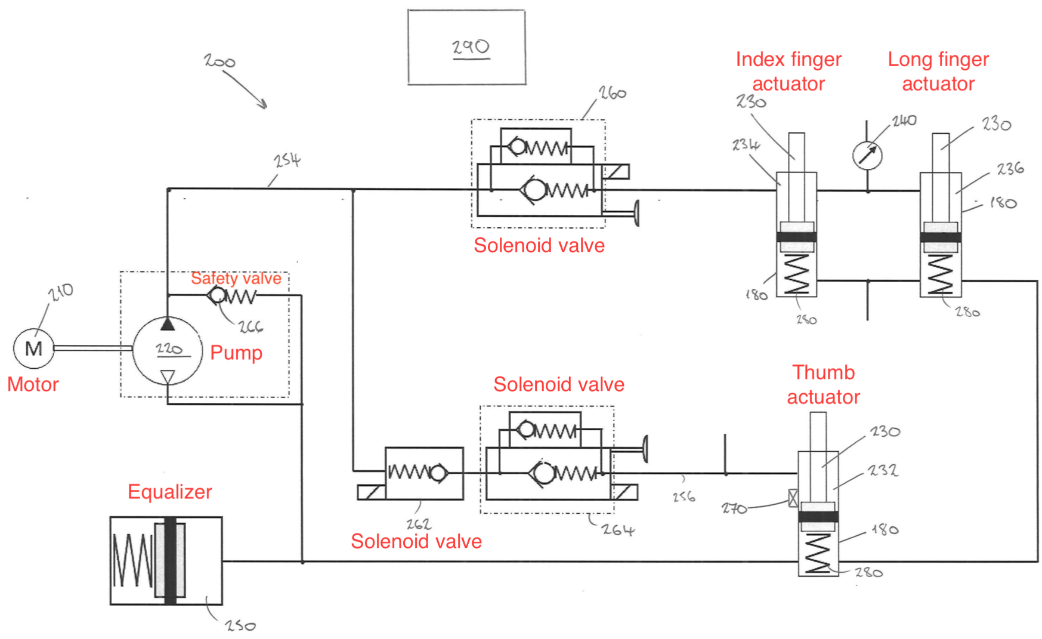

Figure 1.3: Image showing an overview of the different parts of $MyHand^{TM}$. *Credit: Hy5 (2018b).*

In figure 1.3 we can see a schematic overview of the hydralic system of the hand. When the pump is running, it is displacing fluid from the low pressure side of the hand and pushing it into the high-pressure side. The increasing pressure on the high-pressure sides pushes the pistons and closes the hand. To compensade for the loss of fluid volume on the low-pressure side, the system has an equalizer which works as a piston on a spring, making the volume smaller as the pressure gets smaller.

1.3 Literature study

1.3.1 Introduction

This chapter explores some of the literature on mathematical modeling, hydraulics modeling, small-scale hydraulics, and hydraulics in prosthetics.

The aim of the literature study were mainly to find literature which had explored modeling on similar systems (small-scale hydraulics), finding useful resources that could be used for the modeling work in this project. And while the focus were not on gaining a complete and accurate overview of the field, this study does provide some history and context on modeling and small-scale hydraulics for prostethics.

1.3.2 Inclusion criteria

Publications were only included in the analysis if they had to do with mathematical modeling of hyudralic systems and components. Initially I was only going to look at small-scale hydraulic systems, but realized that there are very little literature on this subject, so I had to made the inclusion criteria larger.

1.3.3 Methodology

I used online resources to look for relevant literure. At first, I searched for relevant literature at the local libraries. Then I extended my search by using google scholar. If I found a relevant source, I searched its references for other relevant sources.

The terms I used to search for literature were similar to these:

- Modelling small-scale hydraulics
- Modelling hydraulic systems
- Small-scale hydraulics
- hydraulics in prosthetics
- Mathematical modelling of hydraulic systems
- modelling tiny hydraulics

1.3.4 Modeling hydraulic systems and components

Hydraulics has been around for a long time, although it was not until the 1600s that proper modeling and mathematical dynamics of hydraulic systems were first derived. Among the pioneers were Benedetto Castelli, who in 1619 published *On the measurement of Running Water*, and were a consultant to the Pope on hydraulic projects. Blaise Pascal (1623–1662), studied hydrodynamics and hydrostatics, and discovered the theory behind hydraulics, which led to the invention of the hydraulic press by Joseph Bramah. Another important figure is Daniel Bernoulli, who published the book *Hydrodynamica* in 1738, where the famous Bernoullis principle is described, which subsequently led to bernoullis equation relating a fluids speed to it's pressure. In 1687, Isaac Newton first published the third laws of motion, which forms the foundation of general mathematical modeling on physical systems (Wikipedia (2020*a*)).

Today, the principles formalized by Newton, bernoulli, and pascal, are still used today to form mathematical models of hydraulic systems. During the 1900s, modeling has been widely applied to hydraulic systems in a variety of applications.

In 2003, Olav Egeland and Tommy Gravdahl published the book *Modeling and Simulation for Automatic Control* (Egeland & Gravdahl (2003)). This book was written with the aim to solve a problem: at the time, to get relevant modeling information of a process, you had to use several books and resources, and it took a long time to go through all the material to find the information that suited your process. So the aim of this book is to supply the control engineer with sufficient modeling background to design controllers for a wide range of processes. The material which is included in this book is based on the experiences of the authors and developments in the industry of automatic control. It is now used as course material for the *Modeling and simulation* course at NTNU in Trondheim. This book has a highly useful chapter on modeling hydralics, showing how to form general models of all the components in MyHand.

In 2013, Per I. Bye wrote the book *Hydraulikk* (Bye (2013)), providing a practical approach to describing hydraulics and the fundamental principles, formulas and practical knowledge used in the hydraulic field. Compared to *Modeling and Simulation for Automatic Control*, this book is more basic, providing the practical and theoretical background of hydraulics.

This short list is by no way exhaustive. There exists many books on hydraulics. These are the two sources I found most useful for the modeling in this report.

1.3.5 Small-scale hydraulics

Small-scale hydraulics is a relatively new field which has been made possible due to the recent technological advancements in manufacturing and metal 3D-printing, making it possible to create smaller solid pieces with low tolerances. Since this is a new field, not much literature exists on small-scale hydraulics.

In 2013, Jicheng Xia and WK Durfee investigated small-scale hydraulic and electromechanical actuators. They found that for applications where the output power were less than 100 W, a hydraulic solution would be lighter than an equivalent electromechanical solution only if it runs at high pressure (Xia & Durfee (2013)). Later, in 2015, Xia completed his phD thesis called *Modeling and Analysis of Small-Scale Hydraulic Systems*, where he concluded that for applications where actuator system weight matters the most, high pressure small-scale hydraulic systems are preferred over electro-mechanical systems, but for applications where the overall system weight matters the most, electro-mechanical systems work better (Xia (2015)).

In 2017, Brett Cullen Neubauer published a phD dissertation called *Principles of small-scale hydraulic systems for human assistive machines*, which had the aim of finding foundational design principles for small-scale hydraulics within the theme of human assistive machines. Brett developed a computer algorithm to find design principles for small-scale hydraulics. Here are some of his findings (Neubauer (2017)):

- In general, system operating pressure should be held above 4 MPa to achieve high power and energy densities, as system efficiency quickly drops for lower system pressures.
- Pump controlled systems illustrate significantly higher power and energy densities compared to valve controlled.
- If the required hydraulic energy capacity of the portable hydraulic power supply is above 300 W·hr, the use of an axial piston pump will result in an overall lighter system compared to a gear pump. The shift in pump types can be attributed to the increased influence of pump efficiency and battery weight as the runtime increases. The volumetric efficiencies of the pumps are comparable, but commercial data suggests that axial piston pumps typically

have 10 to 15 percent higher mechanical efficiencies than gear pumps for small displacement pumps. However, if a large displacement (high flow) pump is required for a short period of time, gear pumps are preferred. In this situation the high power density of the pump outweighs the difference in mechanical efficiency.

1.3.6 Hydraulics in prosthetics

One of the earliest studies (non patent) I could find within hydraulics in prosthetics were in 2005, where Schultz et al (Schulz et al. (2005)) presented a prosthetic hand with 15 degrees of freedom, which used lightweight flexible fluid actuators.

Later, in 2009, L. Love et al (Love et al. (2009)) published a study on mesofluidic actuation for fingers and hand prosthetics. The research concludes with the remarks that this system rivals the human hand in terms of size, dexterity and force capacity. Their system used a 4 mm hydraulic cylinder at each joint, which could generate 10 kg of clamping force with 13.8 MPa fluid pressure.

Hy5 is a company that saw the potentials of hydralic prosthetics. They wanted to fill the gap between standard myoelectric grippers and premium bionic-like hand prosteses, and thus created the hydraulic driven prosthesis MyHand, which was commercially released in 2018. Their system differs from the systems from Schultz et al (Schulz et al. (2005)) and L. Love et al (Love et al. (2009)) in that Hy5 only uses one hydraulic actuator for each finger, with one motor for the whole system. This design mimicking realistic hand gripping without requiring one motor per finger, compared to electrical motor driven bionic-like prosthesis. The result is a sturdy, reliable hand with high force output, that can handle five common grip patterns (Bakka et al. (2020)).

1.3.7 Modeling the hydraulic system in

A study directly related to modeling the hydraulic system in $MyHand^{TM}$ were conducted by Hans Alvar Engmark in 2019 (Engmark (2019)). He took the 3D modeling approach, where he used the software COMSOL to create a 3D-model of the pump and a valve, then simulating with finite element analysis and numerical fluid dynamics. The simulations with COMSOL were problematic and Hans concluded that COMSOL were probably not suitable for those types of simulations.

Chapter 2

Theory

This chapter gives the theoretical background for the modeling and simulation chapters in this report. The areas that will be presented are: general theory about fluids, basic hydraulic principles, physical laws, physical relations, and simulation.

2.1 Fluids

2.1.1 Density

All matter has a property of density, which describes the amount of mass per volume. Density is a result of the fraction of atoms and their weight in a given matter. Therefore, density is not constant, but changes with compression and temperature. The symbol used for density is ρ and the unit is $[kg/m^3]$.

2.1.2 Pressure

Pressure is defined as the force per unit area acting perpendicular to a given surface. The symbol used is p and the unit in this report is MPa $[\frac{N}{mm^2}]$.

Static pressure

The static pressure (when the fluid is still) within a fluid depends on the outward pressure acting on the fluid, as well as the weight of the fluid itself. For a tank open to the atmosphere, where the fluid is still, the pressure at a certain point in the fluid would be described as:

$$p = p_{atm} + \rho g h \tag{2.1}$$

Where p_{atm} is the pressure acting on the fluid from the atmosphere, ρ is the density of the fluid, g is the gravitaional acceleration, and h is the distance from the surface of the fluid.

For a still fluid, all points in a horizontal line will have the same pressure.

For a closed hydralic cylinder, where a piston is acting on the fluid from the top and there are no openings for the fluid to escape, the pressure would be:

$$p = \frac{F}{A} + \rho g h \tag{2.2}$$

Where F is the force acting on the piston, and A is the surface area of the piston.

Dynamic pressure

Dynamic pressure is the increase in a moving fluid's pressure over its static pressure due to motion. Another way to think about it is the fluid's kinetic energy per unit volume. Bernoulli's principle describes the dynamic pressure by:

$$p_t - p_s = \frac{1}{2}\rho v^2 \tag{2.3}$$

Where p_t is the total pressure, p_s is the static pressure, and v is the speed of the fluid.

Bernoulli's principle is described in section (2.2.5).

2.1.3 Compression and Bulk Modulus

A fluid, like other forms of matter, can be compressed. Compression means that you are making a volume of matter smaller by adding pressure. *Bulk modulus* is a measure of a matters compressibility, and describes how much a fluids volume will change if the pressure of the fluid changes. The symbol for *Bulk modulus* is β and the unit is the same as for pressure, Pa for example. It is described by this equation:

$$\beta = \frac{pressure}{strain} = \frac{p}{(V_0 - V_n)/V_0} \tag{2.4}$$

Where V_0 is the initial volume and $V_0 - V_n$ is the change in volume. So if the bulk modulus is 35 GPa and the fluid was subjected to a pressure of 0.35 GPa, the fluid would lose one percent of its volume.

2.1.4 Temperature

Temperature is a measure of the kinetic energy of the atoms in a matter. A change in temperature affects a fluids density, viscosity, volume and state. The viscosity of hydraulic oil is often sensitive to temperature and for this reason different oils have different recommended operating temperatures.

2.1.5 Viscocity

The viscosity of a fluid is a measure of its resistance to deformation at a given temperature. A fluid with higher viscosity will be "thicker" and deform more slowly. For example, water has lower viscosity than syrup. In more technical terms, viscosity quantifies the internal friction force between adjacent layers of the fluid.

The viscosity is changing with temperature and pressure. Higher temperature gives a lower viscosity, while a higher pressure gives a higher viscosity. For hydraulics, the changes can be significant and should be considered if the temperature and pressure is changing a lot.

Kinematic viscocity is the definition of viscosity that is most often used in hydraulics, and is defined as the ratio of dynamic viscosity to density:

$$v = \frac{\mu}{\rho} \tag{2.5}$$

Where μ is the dynamic viscosity $[\frac{Ns}{m^2}]$, ρ is the density. *Kinematic viscocity* uses the symbol v and the unit is $[\frac{mm^2}{s}]$.

For a given oil, the *kinematic viscocity* is often described as a curve with viscosity on the y-axis and temperature on the x-axis.

2.2 Flow

2.2.1 Laminar vs turbulent flow

During flow, there always appears friction forces within the fluid. These forces are dependent on the viscosity of the fluid, and the forces make the flow act in different ways. The flow of a fluid can mainly be cathegorized in two types of flow: laminar and turbulent flow.

Laminar flow

At low speeds, the flow is laminar. When a flow is laminar, the speed is biggest in the center of the pipe while almost zero at the pipe wall. The flow is steady, calm, and in parallell layers, meaning that a molecule is flowing in a straight line.

Turbulent flow

When the speed increases, the flow will eventually reach a point where it goes from laminar to turbulent. In turbulent flow, the movement of the molecules becomes random and swirling, no longer in a straight line. The molecules start to interfere with each other and limits eachothers speed. Thus, the flow resistance (and hydraulic pressure losses) becomes larger. This is why turbulent flow is unwanted in hydraulic systems.

The actual point where the flow changes from laminar to turbulent is dependent on viscosity and pipe dimensions.

2.2.2 Reynold's number

The flow condition (laminar, turbulent, or something in between) can be described by the Reynold's number:

$$R_e = \frac{ud}{v} \tag{2.6}$$

Where R_e is the reynolds number, u is the flow speed, and v is the viscosity.

For a pipe system, the flow is generally laminar for $R_e < 2300$. At 2300 (Bye (2013), p. 13), the flow becomes turbulent, and increasingly so as the Reynold number increases.

2.2.3 Viscous friction

A fluid always has a friction against whatever surface it is flowing on, resulting in a friction force. This friction force can be describes with:

$$F_{viscousfriction} = B_{fr}u \tag{2.7}$$

Where B_{fr} is the viscous friction coefficient and u is the speed of the fluid.

Viscous friction coefficient

The viscous friction coefficient depends on the state of the flow, and can be calculated with different formulas depending on the Reynolds number (Bye (2013), p. 14).

For laminar flows ($R_e < 2300$):

$$B_{fr} = \frac{64}{R_e} \tag{2.8}$$

For turbulent flows:

$$B_{fr} = \frac{0.316}{R_e^{0.25}} \tag{2.9}$$

Where R_e is the reynold number.

2.2.4 Pressure losses in pipes

There are always friction between a fluid and the pipe it is flowing in. This causes pressure losses over a length of transmission. The pressure losses are dependent on the friction coefficient, diameter, length of the pipe, density, and speed.

Straight pipes

For straight pipes, the pressure loss is described by the formula (Bye (2013)):

$$\Delta p_{loss} = B_{fr} \frac{l}{d} \rho \frac{v_m^2}{2} \tag{2.10}$$

Where B_{fr} is the friction coefficient, l is the length of the pipe, d is the inner diameter of the pipe, ρ is the density of the fluid, and v_m is the average speed of the flow.

Non-straight pipes

For non-straight pipes, the losses due to the form of the pipe needs to be accounted for. This is described by the formula (Bye (2013)):

$$\Delta p_{loss,form} = \zeta \rho \frac{v_m^2}{2} \tag{2.11}$$

Where ζ is a form resistance coefficient.

The total pressure loss for a non-straight pipe is the combination of friction loss and form loss:

$$\Delta p_{tot} = B_{fr} \frac{l_k}{d} \rho \frac{v_m^2}{2} + \zeta \rho \frac{v_m^2}{2} \tag{2.12}$$

Where l_k is the length of the non-straight pipe.

2.2.5 Bernoullis

Bernoulli's principle describes the relationship between a fluids speed, the position in height, and the total pressure. It can be derived by the principle of conservation of energy (Bye (2013), p.11). The most common form of the principle is this:

$$\frac{v^2}{2} + gz + \frac{p}{\rho} = constant \tag{2.13}$$

Where v is the fluids speed, g is the gravitational acceleration, z is the distance from the top of the fluid, p is the total pressure, and ρ is the density.

2.2.6 Continuity equation

The volume of water flowing through a given position of the pipe will always be the same. If a pipe has different diameters at different positions, the flow Q $[\frac{volume}{time}]$ will be the same in all positions of the pipe. This implies that as the diameter gets smaller, the velocity needs to get bigger in order for the flow to remain the same, and vice versa.

$$q = Av \tag{2.14}$$

$$q_1 = q_2 \tag{2.15}$$

$$A_1 v_1 = A_2 v_2 \tag{2.16}$$

Where q is the flow, A is the cross-section area of the pipe, and v is the speed of the fluid.

2.3 Hydraulic principles

2.3.1 Hydraulic force transmission

Hydraulic force transmission is using the principle of transmitting pressure through a fluid to do work at the end of the transmission line.

For example, consider a closed cylinder with a piston, where the cylinder is connected with a pipe to a pump. As the pump is pushing fluid into the cylinder, the rising pressure inside the cylinder will make the piston move. The force on the piston can be expressed as:

$$F_p = A_p p \tag{2.17}$$

Where F_p is the force acting on the piston, A_p is the area of the piston exposed to the fluid, and p is the pressure of the fluid inside the cylinder.

Another example: Consider two cylinder tanks standing next to each other that are connected at the bottom of each tank with a pipe. If I apply a piston with the same width as the tank and press down on the fluid in tank 1, this pressure generated by this force will be transmitted through the fluid and raise the fluid in tank 2. This lifting force of the fluid in tank 2 can be expressed as (Bye (2013), p. 8):

$$F_2 = \frac{A_2 F_1}{A_1} + A_2 \rho g (h_1 - h_2) \tag{2.18}$$

Where F_2 is the lifting force of the fluid in tank B, F_1 is the applied force with the piston on the surface of tank A, A_2 is the area on the surface of the fluid in tank B, A_1 is the area on the surface of the fluid in tank A, ρ is the density of the fluid, g is the gravitational acceleration, h_1 is the height of the surface in tank A and h_2 is the height of the surface in tank B.

2.3.2 Hydraulic leakage

There are sometimes leakage in a hydraulic system. For example, leakage between between a piston and the cylinder wall, or leakage out of the whole system. The leakage is a result of the pressure differences inside and outside the system. The leakage out of the hydraulic system can be described as:

$$L_e = C_e p \tag{2.19}$$

Where L_e is the external leakage of the system, C_e is the leakage coefficient, and p is the pressure.

There can also be internal leakage, for example in a hydraulic cylinder, where oil leaks from one side of the piston to the other side. This can be described as:

$$L_i = C_i (p_1 - p_2) \tag{2.20}$$

Where L_i is the internal leakage from one side to the other, C_i is the leakage coefficient, p_1 is the pressure on one side of the piston, and p_2 is the pressure on the other side.

The leakage coefficients are often found experimentally, or from previous experiences.

2.3.3 Valves and constrictions

Valves and constrictions narrows the area that the fluid flows through. The flow will stay the same over the valve, but the pressure and speed will change. This can be seen from the continuity equation (2.2.6). It is important to note that the continuity equation assumes incompressible fluid.

A general formula for the flow over valves are (Bye (2013)):

$$q = \alpha A \sqrt{\frac{2}{\rho}(p_1 - p_2)} \tag{2.21}$$

Where α is a valve coefficient which depends on the form of the constriction, A is the area of the constriction, ρ ios the density of the fluid, p_1 is the pressure before the constriction and p_2 is the pressure after the constriction.

2.3.4 Pumps

Pumps works by displacing fluid from one side of the pump to the other, often driven by the torque of a motor. This is a basic general equation for the flow generated by a pump driven by a rotating shaft:

$$q = Dw \tag{2.22}$$

Where q is the flow, D is the pump fluid displacement constant and w is the angular speed of the driving shaft.

2.4 Physical laws and mathematical modeling techniques

2.4.1 Newtons laws of motion

Newton's first law

If the net force of all forces acting on an object is zero, the velocity of the object is constant.

$$\sum F = 0 \Leftrightarrow \frac{dv}{dt} = 0 \tag{2.23}$$

Netwon's second law

The rate of change of momentum of a body is directly proportional to the force applied, and this change in momentum takes place in the direction of the applied force.

$$F = m\frac{dv}{dt} = ma \tag{2.24}$$

Where F is the net force applied, m is the mass, v is the speed and a is the acceleration.

Netwon's second law for rotation

$$\tau = I\frac{dw}{dt} = I\alpha \tag{2.25}$$

Where τ is the torque, I is the moment of inertia, w is the angular speed and α is the angular acceleration.

Netwon's third law

When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.

2.4.2 Mass balance of a hydraulic volume

Mass balance equations are an application of the principle of conservation of mass. In essence, if mass is always conserved within a system, we can form equations that describe the mass flows in and out of this system. For a hydraulic volume V, the mass balance is described by:

$$\dot{m} = w_{in} - w_{out} \tag{2.26}$$

$$\Updownarrow \tag{2.27}$$

$$\frac{d}{dt}(\rho V) = \rho q_{in} - \rho q_{out} \tag{2.28}$$

$$\downarrow \tag{2.29}$$

$$\frac{\dot{\rho} V}{\rho} + \dot{V} = q_{in} - q_{out} \tag{2.30}$$

Where m is the mass of the volume, w_{in} is the mass flow rate into the volume, w_{out} is the mass flow rate out of the volume, ρ is the density of the material in the volume, V is the volume, q_{in} is the flow in, and q_{out} is the flow out.

2.4.3 Relation between pressure and density

Because of the compressibility of fluids, the density ρ is a function of pressure p. A customary assumption is that the relation between the differential $d\rho$ in density and the differential dp in pressure is given by (Egeland & Gravdahl (2003), p. 151):

$$\frac{d\rho}{\rho} = \frac{dp}{\beta} \tag{2.31}$$

Where β is the bulk modulus (see 2.1.3).

This relation is useful when deriving the pressure dynamics of a given volume, since when combined with a mass balance equation, it introduces the derivative of the pressure $\dot{p}$.

2.5 Simulation of a dynamic model

2.5.1 ode45

ode45 is an ODE solver in MatLab, which is commonly used to solve nonstiff differential equations. ode45 is based on an explicit Runge-Kutta (4,5) formula, and uses variable time-step for each integration.

Chapter 3

Method

3.1 Modeling

I start with pen and paper, sitting down and trying to write down and understand the system, continuing until I have a good mathematical description of the system through a mathematical formulation. This part of the process includes making assumptions and simplifying processes inside the system. To my disposal, I have the the mathematical tools and knowledges gathered from the sources in the literature review. Physical laws will be used to derive equations that describes certain aspects of the hydraulic components. I will then combine all the models into a model that describes the whole system.

For the model, a balance need to be struck between complex and accurate, or simple and approximate. In this project, I aim for an initially simple model, which after validation can be made more complex if it is shown to not be accurate enough for it's purposes.

3.2 Simulation

The mathematical model will be entered into MatLab, where I will use the built-in solver ode45 (2.5.1) to simulate the system. Through the simulations I will be able to see how the derived model of the system behaves. I will also be able to adjust the parameters to see how different parameters affect the behavior of the model.

3.2.1 MatLab

MatLab is a software (and programming language) often used for numerical computing and simulation. I will use MatLab to program the mathematical model, and use one of the built in functions, ode45, to simulate the states in a given timeframe and with given parameters.

Chapter 4

Modeling

4.1 Assumptions and simplifications

In order to make the model as simple and easy to manage as possible, several assumptions and simplifications has been made. The simplifications are:

- No impulse delay
- No fluid compression in flow equations
- Solenoid valves are ignored
- No pressure losses from the transmission line
- The pressure in the pump is static
- Linear spring force
- No stiction (static friction)
- No safety valve

These assumptions needs to be challenged at a later point when validating the model against real measurement data from the hand. The arguments for the changes and the consequences for the model will be discussed below.

4.1.1 No impulse delay

Pressure changes are travelling at the speed of sound, which is around $1000\frac{m}{s}$ in hydraulic oil. Since the distances between the components in the hand are so small, the pressure differences in the volumes should even out in less than 1 ms. This delay is negligable, so the pressure is assumed to be the same over each whole volume.

Consequence for the model:

- Pressure is a function of time only (leaving out distance): $p = p(t)$

4.1.2 No fluid compression in the flow equations

The speed of the pressure dynamics in the cylinders are assumed to be much slower than the flow dynamics. Therefore, the compression within the flow dynamics are assumed to be negligable and the flow dynamics can be treated as static equations.

Consequence for the model:

- The flow equations become static, i.e. no time derivative of flow appears: $q_1 = q_2 + q_3$

4.1.3 Solenoid valves are ignored

The solenoid valves are either fully open or closed. Therefore it is assumed that they does not affect the flow much differently than the pipe in the transmission line, and they can therefore be incorporated in the transmission line model.

Consequence for the model:

- The valve is simply removed from the model and treated as part of the transmission line.

4.1.4 No pressure loss in the transmission line

There are always some pressure losses in pipes. However, in the case of MyHand the transmission lines are so short that the initial assumption is that the losses due to the transmission line is negligable. However, due to the small diameter and non-straight form of the pipes, this assumption should be challenged and validated against real pressure data at a later stage. So, while this assumption is likely to be changed later on, it serves well for the first iteration of the model because it makes the model easier to manage.

Consequence for the model:

- With this assumption the model only requires two pressure nodes, since the pressure is the same at the outlet of the pump and each inlet of the cylinders, and the pressure is also the same at each outlet of the cylinders, inlet of the equalizer, and inlet of the pump.

4.1.5 Static pressure model in pump

The assumption is that the volume of the pump is much smaller than the volumes of the other components, such that the pressure dynamics in the pump is negligable compared to the dynamics in the other volumes.

Consequence for the model:

- The pressure in the pump is treated as a static rather than dynamic. The model for the pump therefore only includes dynamic rotation and static flow equation.

4.1.6 Linear spring force

In reality, the force from a spring is not linear with the displacement. The assumption is that a linear model of the spring is sufficiently close to reality for this model.

Consequence for the model:

- By using a linear model, the spring can be described with Hooke's law: $F_{spring} = kx$. Where x is the displacement from the springs resting positon, and k is a constant factor that describes the stiffness of the spring.

4.1.7 No stiction for cylinder pistons

Stiction is the static friction that needs to be overcome to enable motion of an object. It is assumed that this effect does not hugely affect the model, and therefore for the sake of simplification, stiction will not be included in this first iteration of the model.

Consequence for the model:

- The model and simulation only needs one equation for the friction. With stiction, the model would need to switch between two equations of friction by detecting an event in the simulation. The event being when a certain friction force treshold is reached.

4.1.8 No safety valve

The safety valve opens when the pressure gets too high on the high-pressure side. For a simulation, it is of interest to simulate pressures that may be higher than the safety valve allows, therefore this valve will not be modeled.

Consequence for the model:

- The model will not include a safety valve, enabling the pressure to rise indefinitely.

4.2 Modeling the hydraulic components of MyHand 2

Symbol	**Meaning**
ρ	Density (kg/mˆ3)
β	Bulk modulus
F_{spring}	Force from spring
F_p	Force from piston
F_L	Force from load (from fingers)
p	Pressure
V	Volume
q	Flow (vol/time)
A	Area
x_p	Position of piston
v_p	Speed of piston
m_p	Mass of piston
B_{fr}	Viscous friction coefficient
D_u	Displacement of the pump
J_u	Rotational inertia of the pump shaft
w_u	Angular velocity of pump gear shaft
T_{em}	Torque from the motor

Table 4.1: The symbols used in the modeling chapter, and their meaning.

Based on the assumptions and simplifications (4.1), the hydraulic components which will be modeled, are:

- Pump
- Cylinders
- Equalizer

These will be modeled one by one, deriving equations describing the dynamic pressure, flow, and movements of the cylinder pistons. The motor driving the pump will not be modeled. The transmission line and solenoid valves are important parts which should eventually be modelled, but for the simplified model in this report they are left out.

The variables used in this chapter is explained in table 4.1.

4.2.1 Pump

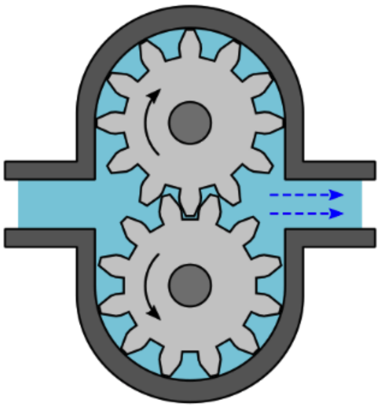

Figure 4.1: Illustration of a general hydraulic pump. *Credit: Savre (2020).*

The pump is driven by a torque from the motor. The torque drives the gears in the pump, which in turn moves oil from the inlet to the outlet. How much fluid the pump displaces is defined by the dispacement constant D_u $[\frac{vol}{rad}]$, and depends on the dimensions of the gears.

Flow

The flow at the inlet and outlet of the pump is a result of the rotational speed of the gears and the displacement constant:

$$q = D_u w_u \tag{4.1}$$

Where w_u is the angular velocity of the gears and D_u is the pump displacement contstant.

Rotational dynamics

It is desirable to gain a dynamic model for w_u. Using Newton's second law for rotation (2.4.1), we get the following equations for the angular motion of the driving shaft of the pump:

$$J_u \dot{w_u} = \sum_i \tau_i = -B_{fr} w_u - D_u(p_{outlet} - p_{inlet}) + T_{em} \tag{4.2}$$

Where J_p is the moment of inertia of the pump, w_u is the angular velocity, B_{fr} is the viscuous friction coefficient, D_u is the fluid-displacement of the pump, p_1 is the pressure on the outlet side of the pump, p_2 is the pressure on the inlet side of the pump, and T_{em} is the torque coming from the electric motor.

T_{em} acts as an input, while $-B_{fr} w_u$ and $-D_u(p_{outlet} - p_{inlet})$ acts as a dampening effect. Since the hydraulic system is closed towards the cylinders in MyHand2, the outlet pressure builds up as the pump runs, eventually building enough pressure to stop the pump.

Resulting model

So, we have the dynamic model describing angular velocity of the pump shaft and the resulting flow out of the pump:

$$\dot{w_u} = \frac{-B_{fr}w_u - D_u(p_1 - p_2) + T_{em}}{J_u} \tag{4.3}$$

$$q = D_u w_u \tag{4.4}$$

Where T_{em} serves as an input to the dynamic model.

4.2.2 Hydraulic cylinder

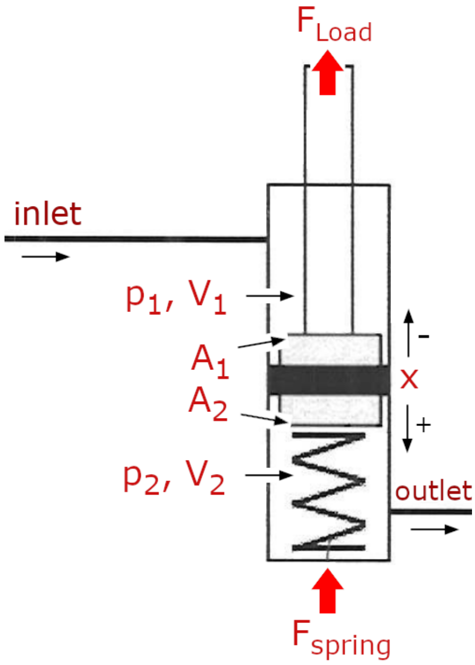

Figure 4.2: Illustration of the cylinder component, denoting variables used in the derivation of the model.

The piston in the cylinder is driven by the difference of pressures on either side of the piston. The pressures increases or decreases because of the flow which goes in our out of the volumes in the cylinder. A model for the dynamic pressure and flow will be derived, as well as motion equations for the piston.

Pressure dynamics

The pressure dynamics in a hydraulic cylinder can be modeled by combining the mass balance principle (2.4.2) with the relation between pressure and density (2.4.3).

From the mass balance principle (2.4.2), we have that the mass balance for a hydralic volume can be described by:

$$\frac{\dot{\rho}V}{\rho} + \dot{V} = q_{in} - q_{out} \tag{4.5}$$

And from the relation between pressure and density (2.4.3), we have that:

$$\frac{d\rho}{\rho} = \frac{dp}{\beta} \tag{4.6}$$

Putting equation 4.6 into equation 4.5, we get the following equation for the pressure dynamics in a volume:

$$\frac{V}{\beta}\dot{p} + \dot{V} = q_{in} - q_{out} \tag{4.7}$$

$$\downarrow \tag{4.8}$$

$$\dot{p} = \frac{\beta}{V}(q_{in} - q_{out} - \dot{V}) \tag{4.9}$$

In the cylinder we have two volumes: V_1 and V_2, as denoted in figire 4.2. The volumes can be described by:

$$V_1 = V_{1,0} + A_1 x_p \tag{4.10}$$

$$V_2 = V_{2,0} - A_2 x_p \tag{4.11}$$

$$\dot{V_1} = A_1 v_p \tag{4.12}$$

$$\dot{V_2} = -A_2 v_p \tag{4.13}$$

Where $V_{i,0}$ is the initial volume of volume i, A_i is the area of the piston on the $i'th$ side of the piston, x_p is the position of the piston in the cylinder, v_p is the speed of the piston, and i is 1 or 2, denoting the two sides of the cylinder. Note that x_p is positive in the direction which makes volume one larger (and volume two smaller).

Combining the equations for the volumes (eq. 4.10 - 4.13) with the pressure dynamics (eq. 4.9) for the volumes on either side of the piston, we arrive at the two equations below, describing the pressure dynamics at both sides of the piston in the cylinder:

$$\dot{p_1} = \frac{\beta}{V_{1,0} + A_1 x_p}(q_{1,in} - \cancel{q_{1,out}} - A_1 v_p) \tag{4.14}$$

$$\dot{p_2} = \frac{\beta}{V_{2,0} - A_2 x_p}(\cancel{q_{2,in}} - q_{2,out} + A_2 v_p) \tag{4.15}$$

Where p_1 is the pressure on the top side of the cylinder (high pressure side), and p_2 is the pressure on the "bottom" side of the cylinder (low pressure side), separated by the piston. Note that $q_{1,out}$ and $q_{2,in}$ are zero.

Leakage would normally have to be accounted for in this model, but in MyHand 2 there are no leakages.

Equations of motion for the piston

The pressure dynamics are dependent on a proper model of the movement of the piston. We will therefore derive the equations of motion for the piston in the cylinder.

From newton's second law of motion (2.4.1), we get the following equation for the piston:

$$m_p \dot{v_p} = \sum F = A_1 p_1 - A_2 p_2 - B_{fr} v_p - F_L - F_{spring} \tag{4.16}$$

Where A_1 and A_2 is the area of the piston on either side, p_1 and p_2 are the pressures on either side, B_{fr} is the viscuous friction coefficient, v_p is the speed of the piston, F_L is the load force (from closing the fingers around something), and F_{spring} is the force from the spring in the cylinder.

Resulting model

The resulting model describes the motion of the piston as well as the pressure dynamics on both sides of the cylinder:

$$\dot{p_1} = \frac{\beta}{V_{1,0} + A_1 x_p}(q_{1,in} - A_1 v_p) \tag{4.17}$$

$$\dot{p_2} = \frac{\beta}{V_{2,0} - A_1 x_p}(A_2 v_p - q_{2,out}) \tag{4.18}$$

$$\dot{v_p} = \frac{A_1 p_1 - A_2 p_2 - B_{fr}\dot{x_p} - F_L + F_{spring}}{m_p} \tag{4.19}$$

$$\dot{x_p} = v_p \tag{4.20}$$

4.2.3 Equalizer

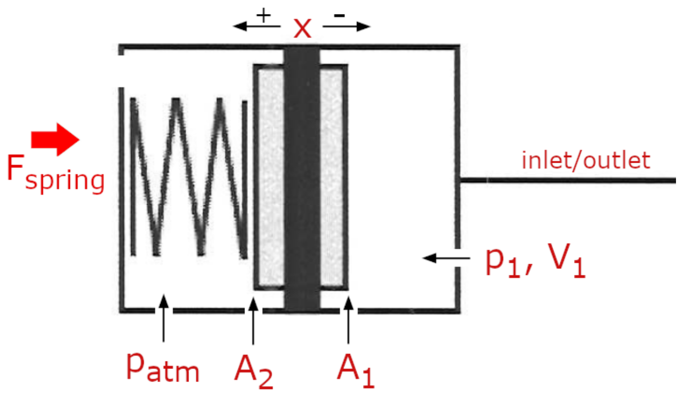

Figure 4.3: Illustration of the Equalizer component, denoting variables used in the derivation of the model.

The purpose of the equalizer is to automatically adjust the pressure on the low-pressure side of the system. The equalizer is a cylinder with a piston on a spring, which movement is driven by the change of pressure in the system. The left chamber of the equalizer is open to the atmosphere, so the pressure on that side is simply the atmospheric pressure. The right side can be modelled with with the same principles as the cylinders.

Pressure dynamics

For the inlet side of the equalizer, we use the same principle for the pressure dynamics as with the cylinders (4.2.2):

$$\dot{p} = \frac{\beta}{V}(q_{in} - q_{out} - \dot{V}) \tag{4.21}$$

Where the equations for the volumes are:

$$V_1 = V_{1,0} + A_1 x_p \dot{V_1} = A_1 v_p \tag{4.22}$$

Piston motion

For the piston motion, we use the same principles as for the cylinders (4.2.2), but without the load force from the fingers, and with p_2 being replaced with a constant atmospheric pressure p_{atm}:

$$m_p \dot{v_p} = A_1 p_1 - A_2 p_{atm} - B_{fr} v_p - F_{spring} \tag{4.23}$$

Resulting model

The following equations describe the pressure dynamics and motion of the piston for the equalizer:

$$\dot{p_1} = \frac{\beta}{V_{1,0} + A_1 x_p}(q_{1,in} - A_1 v_p) \tag{4.24}$$

$$\dot{v_p} = \frac{A_1 p_1 - A_2 p_{atm} - B_{fr} v_p - F_{spring}}{m_p} \tag{4.25}$$

$$\dot{x_p} = v_p \tag{4.26}$$

4.3 Combining components into a model of the full system

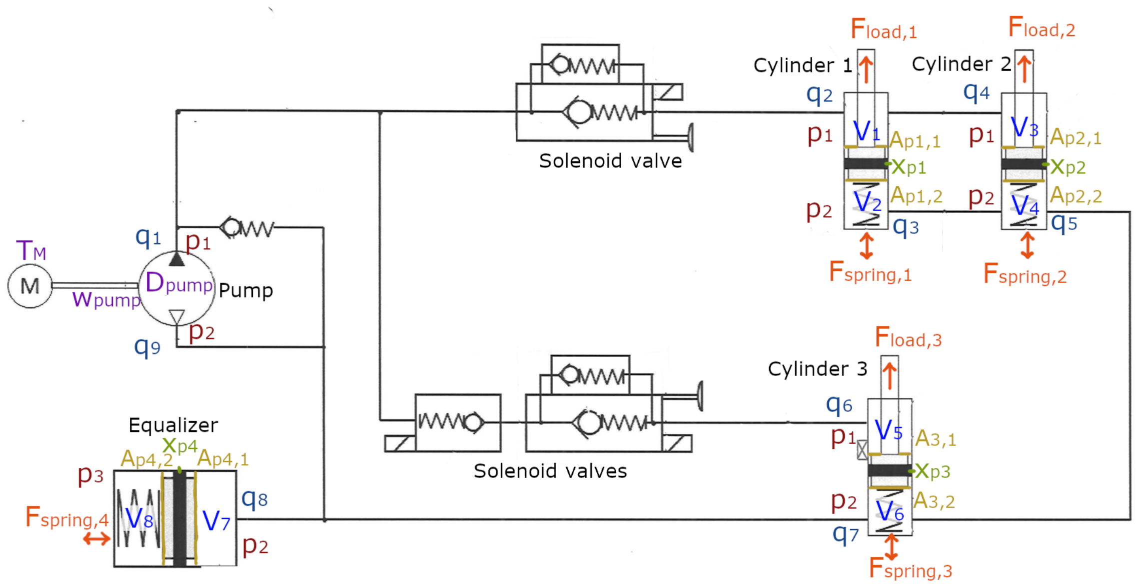

Figure 4.4: Overview of the simplified system and the parameters used in the model.

Symbol	Meaning
p_i	Pressure node i
V_i	Volume i
$V_{i,0}$	initial volume of volume i
q_i	Flow in position i (vol/time)
$A_{pi,1}$, $A_{pi,2}$	Area of piston side 1 and 2, in cylinder i
x_{pi}	Position of piston inside cylinder i (relative to start position)
v_{pi}	Speed of piston i
m_{pi}	Mass of piston i
$F_{load,i}$	Force from finger i (e.g. the finger holds on to something)
$F_{spring,i}$	Force from the spring in cylinder i
k_i	Spring coefficient for spring i
D_u	Displacement of the pump
J_u	Rotational inertia of the pump shaft
w_u	Angular velocity of the pump shaft
B_{fr}	Viscuous friction coefficient
β	Bulk modulus
T_m	Torque from the motor

Table 4.2: The symbols used in the overview figure and in the combined model.

In this chapter, the modeled components will be compined in to a combined dynamic model of the system. Figure 4.4 shows an overview of the system and the variables used in the model.

Due to the assumptions (4.1) that there are no friction in the transmission lines, and the valves are ignore, there are only two pressure nodes, this will be discussed below.

Note that in this section the variable names have changed compared to section 4.2 where each component were modeled individually. Use the overview (fig. 4.4) to see what the variable names represent in this section.

4.3.1 Combined pressure dynamics

Due to the assumtions discussed earlier (4.1), the pressure will be the same in the whole high-pressure side of the system, and the same for the low-pressure side. This results in a system model with only two pressure nodes, as can be seen in the overview in figure 4.4.

This means that with our pressure dynamic model, the volumes on each side will be combined into one volume respectively:

$$V_{tot,1} = V_{1,0} + V_{3,0} + V_{5,0} + A_{p1,1}x_{p1} + A_{p2,1}x_{p2} + A_{p3,1}x_{p3} \tag{4.27}$$
$$V_{tot,2} = V_{2,0} + V_{4,0} + V_{6,0} + V_{7,0} - A_{p1,2}x_{p1} - A_{p2,2}x_{p2} - A_{p3,2}x_{p3} + A_{p4,1}x_{p4} \tag{4.28}$$
$$\dot{V_{tot,1}} = A_{p1,1}v_{p1} + A_{p2,1}v_{p2} + A_{p3,1}v_{p3} \tag{4.29}$$
$$\dot{V_{tot,2}} = -A_{p1,2}v_{p1} - A_{p2,2}v_{p2} - A_{p3,2}v_{p3} + A_{p4,1}v_{p4} \tag{4.30}$$
$$\tag{4.31}$$

Where $V_{tot,1}$ is the combined volume on the high-pressure side, and $V_{tot,2}$ is the combined volume on the low-pressure side.

The flow into and out of these volumes become equal to the flow from the pump:

$$q_1 = w_u D_u \tag{4.32}$$
$$q_2 = -w_u D_u \tag{4.33}$$

Where q_1 is the flow into the high-pressure side, and q_2 is the flow out of the low-pressure side.

The combined pressure dynamics becomes:

$$\dot{p_1} = \frac{\beta}{V_{tot,1}}(q_1 - \dot{V_{tot,1}}) \tag{4.34}$$
$$\dot{p_2} = \frac{\beta}{V_{tot,2}}(q_2 - \dot{V_{tot,2}}) \tag{4.35}$$
$$\tag{4.36}$$

4.3.2 Everything together

With the combined pressure dynamics and replacing F_{spring} with kx, we get the following combined dynamics:

$$V_{tot,1} = V_{1,0} + V_{3,0} + V_{5,0} + A_{p1,1}x_{p1} + A_{p2,1}x_{p2} + A_{p3,1}x_{p3} \tag{4.37}$$
$$V_{tot,2} = V_{2,0} + V_{4,0} + V_{6,0} + V_{7,0} - A_{p1,2}x_{p1} - A_{p2,2}x_{p2} - A_{p3,2}x_{p3} + A_{p4,1}x_{p4} \tag{4.38}$$
$$\dot{V_{tot,1}} = A_{p1,1}v_{p1} + A_{p2,1}v_{p2} + A_{p3,1}v_{p3} \tag{4.39}$$
$$\dot{V_{tot,2}} = -A_{p1,2}v_{p1} - A_{p2,2}v_{p2} - A_{p3,2}v_{p3} + A_{p4,1}v_{p4} \tag{4.40}$$
$$q_1 = w_u D_u \tag{4.41}$$
$$q_2 = -w_u D_u \tag{4.42}$$
$$\dot{w_u} = \frac{-B_{fr}w_u - D_u(p_1 - p_2) + T_m}{J_u} \tag{4.43}$$
$$\dot{p_1} = \frac{\beta}{V_{tot,1}}(q_1 - \dot{V_{tot,1}}) \tag{4.44}$$
$$\dot{p_2} = \frac{\beta}{V_{tot,2}}(q_2 - \dot{V_{tot,2}}) \tag{4.45}$$
$$p_3 = p_{atm} \tag{4.46}$$
$$\dot{v_{p1}} = \frac{A_{p1,1}p_1 - A_{p1,2}p_2 - B_{fr}v_{p1} - F_{load,1} - k_1x_{p1}}{m_{p1}} \tag{4.47}$$
$$\dot{v_{p2}} = \frac{A_{p2,1}p_1 - A_{p2,2}p_2 - B_{fr}v_{p2} - F_{load,2} - k_2x_{p2}}{m_{p2}} \tag{4.48}$$
$$\dot{v_{p3}} = \frac{A_{p3,1}p_1 - A_{p3,2}p_2 - B_{fr}v_{p3} - F_{load,3} - k_3x_{p3}}{m_{p3}} \tag{4.49}$$
$$\dot{v_{p4}} = \frac{A_{p4,1}p_2 - A_{p4,2}p_3 - B_{fr}v_{p4} - k_4x_{p4}}{m_{p4}} \tag{4.50}$$
$$\dot{x_{p1}} = v_{p1} \tag{4.51}$$
$$\dot{x_{p2}} = v_{p2} \tag{4.52}$$
$$\dot{x_{p3}} = v_{p3} \tag{4.53}$$
$$\dot{x_{p4}} = v_{p4} \tag{4.54}$$

4.3.3 Modeling when the pistons hits the end of each cylinder

When the pistons hits the end of each cylinder, an opposive reactive force is added to the model, resulting in a net zero acceleration, and zero speed:

$$\dot{v_{p1}} = 0 \tag{4.55}$$
$$\dot{v_{p2}} = 0 \tag{4.56}$$
$$\dot{v_{p3}} = 0 \tag{4.57}$$
$$\dot{v_{p4}} = 0 \tag{4.58}$$
$$v_{p1} = 0 \tag{4.59}$$
$$v_{p2} = 0 \tag{4.60}$$
$$v_{p3} = 0 \tag{4.61}$$
$$v_{p4} = 0 \tag{4.62}$$

This scenario is a case for the simulation, and will be discussed more in the simulation chapter.

Chapter 5

Simulation

The simulation was done using MatLab, by constructing the dynamics as a function and integrating the dynamics with the built-in MatLab function ode45 (2.5.1) over a given time period and with given initial state values.

At this stage it is not important that the parameters are entirely accurate, since we only want to discover if the general characteristic of the simulation makes sense. Therefore the parameters used in the simulation include both real values and educated guesses. When the model is eventually validated against data from the real hand, it is more important to have accurate parameters.

The code for the simulation, dynamics, and events, can be found in the appendix A, B and C.

5.1 Parameters and initial values

Tables 5.1 - 5.5 shows the parameters used for the simulation. Some parameters are changed for different simulations to show different behaviors of the model. These will be discussed along with the plots. The source for all parameters can be seen in the tables.

The initial values for the states in the model can be seen in table 5.6.

General system parameters			
Parameter	**Variable name**	**Value**	**Source**
Diameter of transmission pipes	d	2.5mm	CAD file (Hy5 (2018*a*))
Reynold number	Re	7.795	$R_e = \frac{vd}{v}$ (2.2.2)
Viscous friction coefficient	B_{fr}	8.21 mm^3	$B_{fr} = \frac{64}{R_e}$ for $R_e < 2300$ (2.2.3)
Bulk modulus	β	1000 MPa	Within the standard range for mineral hydraulic oils (Bye (2013), p. 31)
Motor torque	T_m	2600 Nmm	Guess (This is a high value, but it is chosen in order to generate an RPM which reflects the RPM from a pump test sheet from Hy5 (**??**))
Pump displacement	D_u	20 mm^3	Guess
Mass of pump rod	m_{rod}	0.02 kg	Guess
Radius of pump rod	r_{rod}	2.5	Guess
Moment of inertia of pump rod	J_u	0.938 $kgmm^2$	$J_u = \frac{1}{2}m_{rod}r_{rod}^2$ (Wikipedia (2020*b*))

Table 5.1: Table showing general parameters for the hydraulic system, which were used in the simulation.

Cylinder 1 (index finger)			
Parameter	**Variable name**	**Value**	**Source**
Load force on finger 1	$F_{load,1}$	20 N	Within the operating range (Hy5 (2020))
Spring constant in cylinder 1	k_1	1.53 N/mm	Hy5 repr.
Area on inlet side of piston in cylinder 1	$A_{p1,1}$	119.7 mm^2	CAD file (Hy5 (2018*a*))
Area on outlet side of piston in cylinder 1	$A_{p1,2}$	154 mm^2	CAD file (Hy5 (2018*a*))
Mass of piston in cylinder 1	m_{p1}	0.02 kg	Guess
Initial volume of the inlet side in cylinder 1	$V_{p1,1,init}$	4370.5 mm^3	CAD file (estimate) (Hy5 (2018*a*))
Initial volume of the outlet side in cylinder 1	$V_{p1,2,init}$	10196 mm^3	CAD file (estimate) (Hy5 (2018*a*))
Limit of piston movement in cylinder 1	$x_{p1,limit}$	58.22 mm	CAD file (estimate) (Hy5 (2018*a*))

Table 5.2: Table showing the simulation parameters for cylinder 1 (index finger)

Cylinder 2 (long finger)			
Parameter	**Variable name**	**Value**	**Source**
Load force on finger 2	$F_{load,2}$	20 N	Within the operating range (Hy5 (2020))
Spring constant in cylinder 2	k_2	1.53 N/mm	Hy5 repr.
Area on inlet side of piston in cylinder 2	$A_{p2,1}$	119.7 mm^2	CAD file (Hy5 (2018a))
Area on outlet side of piston in cylinder 2	$A_{p2,2}$	154 mm^2	CAD file (Hy5 (2018a))
Mass of piston in cylinder 2	m_{p2}	0.02 kg	Guess
Initial volume of the inlet side in cylinder 2	$V_{p2,1,init}$	4370.5 mm^3	CAD file (estimate) (Hy5 (2018a))
Initial volume of the outlet side in cylinder 2	$V_{p2,2,init}$	10196 mm^3	CAD file (estimate) (Hy5 (2018a))
Limit of piston movement in cylinder 2	$x_{p2,limit}$	58.22 mm	CAD file (estimate) (Hy5 (2018a))

Table 5.3: Table showing the simulation parameters for cylinder 2 (long finger)

Cylinder 3 (Thumb)			
Parameter	**Variable name**	**Value**	**Source**
Load force on thumb	$F_{load,3}$	20 N	Within the operating range (Hy5 (2020))
Spring constant in cylinder 3	k_3	2.36 N/mm	Hy5 repr.
Area on inlet side of piston in cylinder 3	$A_{p3,1}$	78.9 mm^2	CAD file (Hy5 (2018a))
Area on outlet side of piston in cylinder 3	$A_{p3,2}$	113.1 mm^2	CAD file (Hy5 (2018a))
Mass of piston in cylinder 3	m_{p3}	0.02 kg	Guess
Initial volume of the inlet side in cylinder 3	$V_{p3,1,init}$	2703.4 mm^3	CAD file (estimate) (Hy5 (2018a))
Initial volume of the outlet side in cylinder 3	$V_{p3,2,init}$	6308.7 mm^3	CAD file (estimate) (Hy5 (2018a))
Limit of piston movement in cylinder 3	$x_{p3,limit}$	51.94 mm	CAD file (estimate) (Hy5 (2018a))

Table 5.4: Table showing the simulation parameters for cylinder 3 (thumb).

Equalizer parameters			
Parameter	**Variable name**	**Value**	**Source**
Spring constant in equalizer	k_4	4.51 N/mm	Hy5 repr.
Area on inlet side of piston in equalizer	$A_{p4,1}$	289.5 mm^2	CAD file (Hy5 (2018a))
Area on outlet side of piston in equalizer	$A_{p4,2}$	255 mm^2	CAD file (Hy5 (2018a))
Mass of piston in equalizer	m_{p4}	0.02 kg	Guess
Initial volume of the inlet side in equalizer	$V_{p4,1,init}$	30468 mm^3	CAD file (estimate) (Hy5 (2018a))
Atmospheric pressure on the outside of equalizer	p_{atm}	0.1013 MPa	Open to air
Limit of piston movement in equalizer	$x_{p4,limit}$	110.2 mm	CAD file (estimate) (Hy5 (2018a))

Table 5.5: Table showing the simulation parameters for the equalizer.

States			
State	**variable name**	**Initial value**	**Source**
Pump angular velocity	w_u	0	
Pressure on inlet /high pressure side	p_1	2 Bar	Hy5 repr.
Pressure on outlet /low pressure side	p_2	2 Bar	Hy5 repr.
Piston 1 position (index finger cylinder)	x_{p1}	0	
Piston 2 position (long finger cylinder)	x_{p2}	0	
Piston 3 position (thumb cylinder)	x_{p3}	0	
Piston 4 position (equalizer)	x_{p4}	17 mm	Testing in simulation
Piston 1 speed	v_{p1}	0	
Piston 2 speed	v_{p2}	0	
Piston 3 speed	v_{p3}	0	
Piston 4 speed	v_{p4}	0	

Table 5.6: Table showing the states and their initial values in the simulations.

5.2 Events and model changes during simulation

In order to simulate when the pistons reach the end of each cylinder, event detection was added in MatLab through ode45. The events were triggered when x_p reached the limit length of the cylinder. When the event triggers, the simulation restarts with a new initial value for the speed of the piston $v_p = 0$, and the model for the motion of the piston adds an opposing reacting force which indicates that the piston is now pushing against a "wall", resulting in $\dot{v_p} = 0$. The dynamic function detects whether the piston is pushing against the "wall" or not by calculating the sum of the forces on the piston and seeing if the sum force is positive or negative. If the sum force is again negative, the opposing reacting force is removed, simulating that the piston is moving backwards from the "wall".

The same type of event is detected and applied when the piston reaches the start of the cylinder.

See appendix A, B and C for the implementation of these events and model changes.

5.3 Simulating closing and opening of the hand

To simulate the closing and opening of the hand, motor torque parameter were changed during the simulation and a one-way valve were emulated. This were implemented using events. So when the simulation detected that all fingers had reached their closed position, the motor torque were turned to zero and the one-way valve were activated by turning the pressure dynamics to zero within the cylinders.

See appendix A and C for the implementation.

5.4 Simulations

* Standard setting * hold closed * close and open * Lower Bulk modulus * hold closed * Low viscous friction * hold closed

Presented below are several simulations with different times and parameters. The subsections title indicates the parameter which is special for that specific simulation.

Please see table 5.6 as a reference for the meaning for the variables in the plots below.

5.4.1 Standard setting

Standard in this context mean that the parameters are as close to the real values as could be estimated, as shown in tables 5.1 - 5.5.

Closing the hand

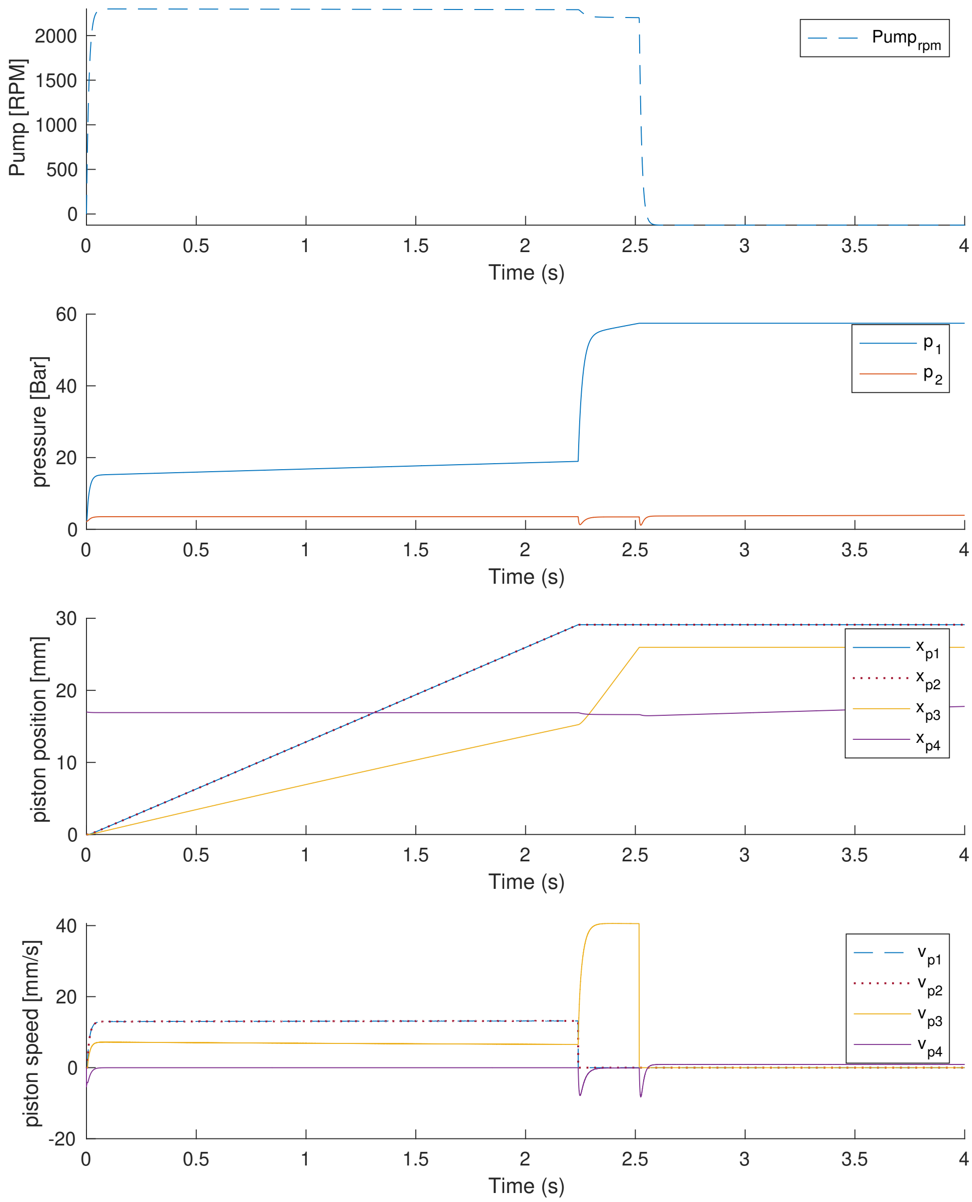

Figure 5.1: Plot of simulation over 4 seconds, with standard parameters and closing of the hand. The plot shows the changes in pump rpm, pressure, piston position and piston speed.

We can see in figure 5.1 that the pump quickly goes up to around 2300 rpm, resulting in a movement of the pistons in the cylinders (v_{p1}, v_{p2} and v_{p3}). The position of the piston in the equalizer starts with a position at 17mm and moves slightly backwards at the start (as can be seen at v_{p4}), thus quickly equalizing the pressure at the low-pressure side. When piston one and two first reaches the end of the cylinders at 2.4 seconds, we can see how the rate of pressure for p_1 and the speed of piston three v_{p3} increases. We can see that the movement of the pistons in cylinder one and two matches each other exactly. This is a result of the simplified dynamic model where all the volumes of all the cylinders on either pressure side were combined into one, and since cylinder one and two are identical, the equation of motion becomes the same, generating this result. When all three pistons in the cylinders has reached the end at 2.5 seconds, the torque to the pump is turned off and the closing of the valve to the cylinders is simulated by stabilizing the pressure p_1 with $\dot{p_1} = 0$. The pressure on the low-pressure side is stable during the simulation because of the equalizer and the decreasing of the volume due to piston movements.

The fast pressure dynamics increases very rapidly after the two fingers has reached the end of their cylinders. This is a result of the bulk modulus being so high. In our model, the pressure dynamics is directly proportional to the value of the bulk modulus: $\dot{p} = \frac{\beta}{V}(q_{in} - q_{out} - \dot{V})$

Closing and opening the hand

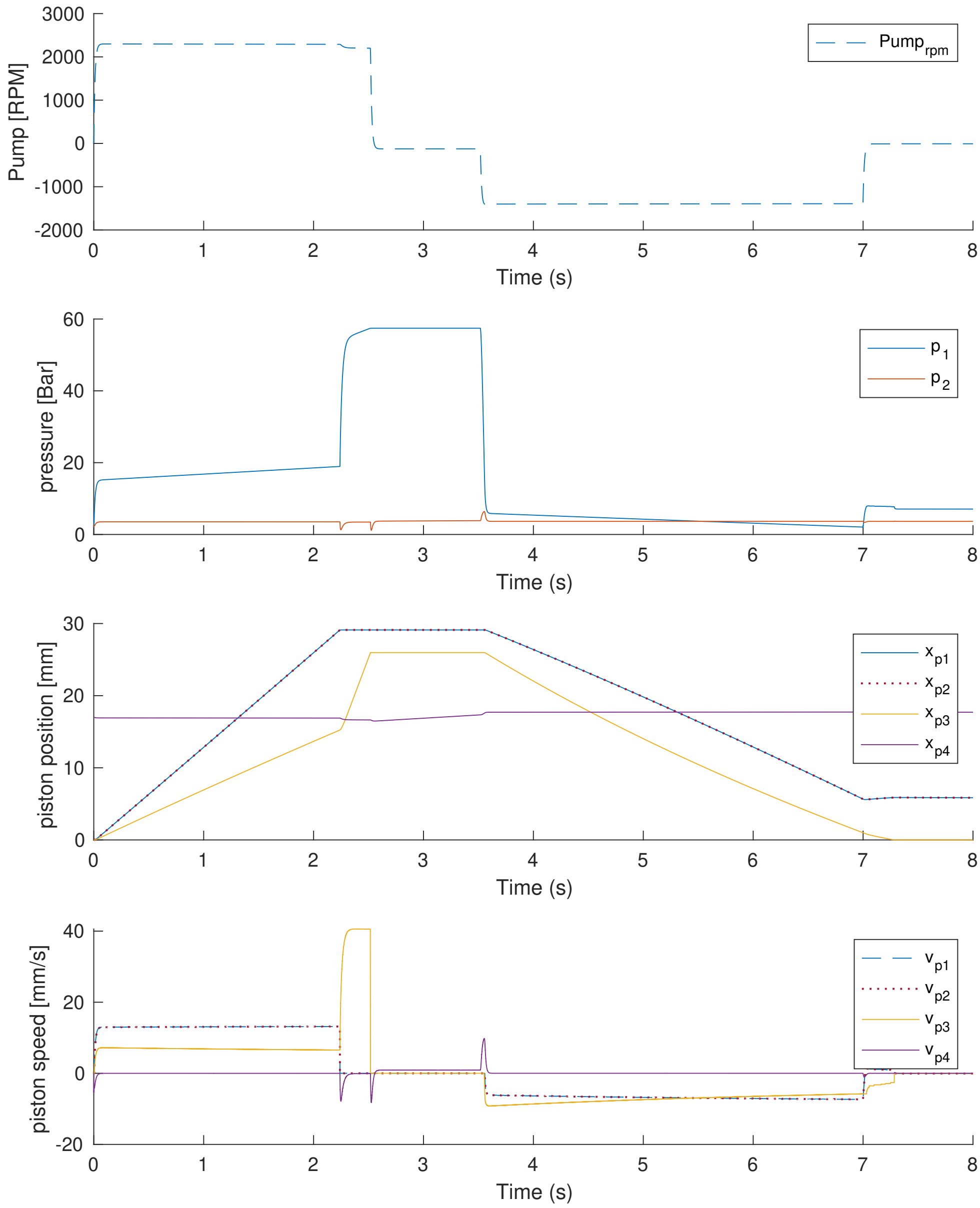

Figure 5.2: Plot of simulation over 8 seconds, with standard parameters and closing and opening of the hand. The plot shows the changes in pump rpm, pressure, piston position and piston speed.

In figure 5.2, closing and opening of the hand is simulated. When the hand reaches the closed position at 2.5 seconds, it holds the grip for one second by turning the motor torque to zero and simulating the closing of the valve ($\dot{p_1} = 0$). At 3.6 seconds, the motor torque T_m is reversed and the valve is opened ($\dot{p_1}$) uses the normal dynamics), making the pump run backwards and decreasing the pressure on the high-pressure side. This makes the cylinder piston reverse direction, simulating the opening of the hand. When the hand is open, the torque of the motor is again turned to zero.

This simulation of closening and opening the hand is not perfect. I had to turn of the motor when the thumb reached the start position. If I waited to turn of the pump until all fingers had reached the start position, fast pressure dynamics (due to the high bulk modulus) made the pressure p_1 decrease fast, becoming negative. This is a limitation of the simulation of closening and opening the hand.

5.4.2 Bulk modulus = 10 MPa (Standard: $\beta = 1000 Mpa$)

In figures 5.3 and 5.4 we can see how the dynamics works with a lower bulk modulus. This simulates a hydraulic fluid which is much more compressible, making the pressure dynamics slower. This does not reflect a practical hydraulic oil, but provides an interesting view of the model with this type of fluid.

Closing the hand

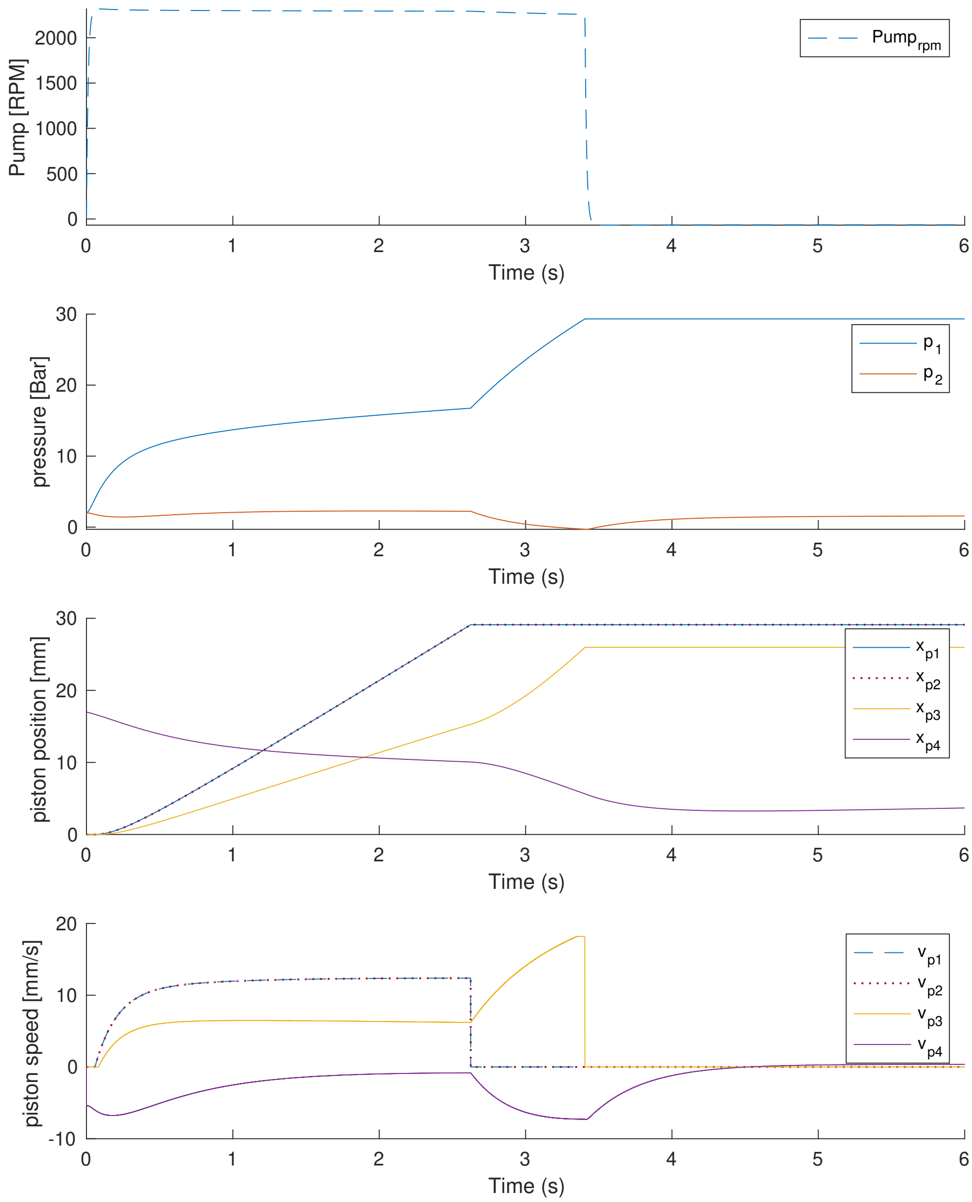

Figure 5.3: Plot of simulation over 6 seconds, with $\beta = 10Mpa$, and closing of the hand. The plot shows the changes in pump rpm, pressure, piston position and piston speed.

Closing and opening the hand

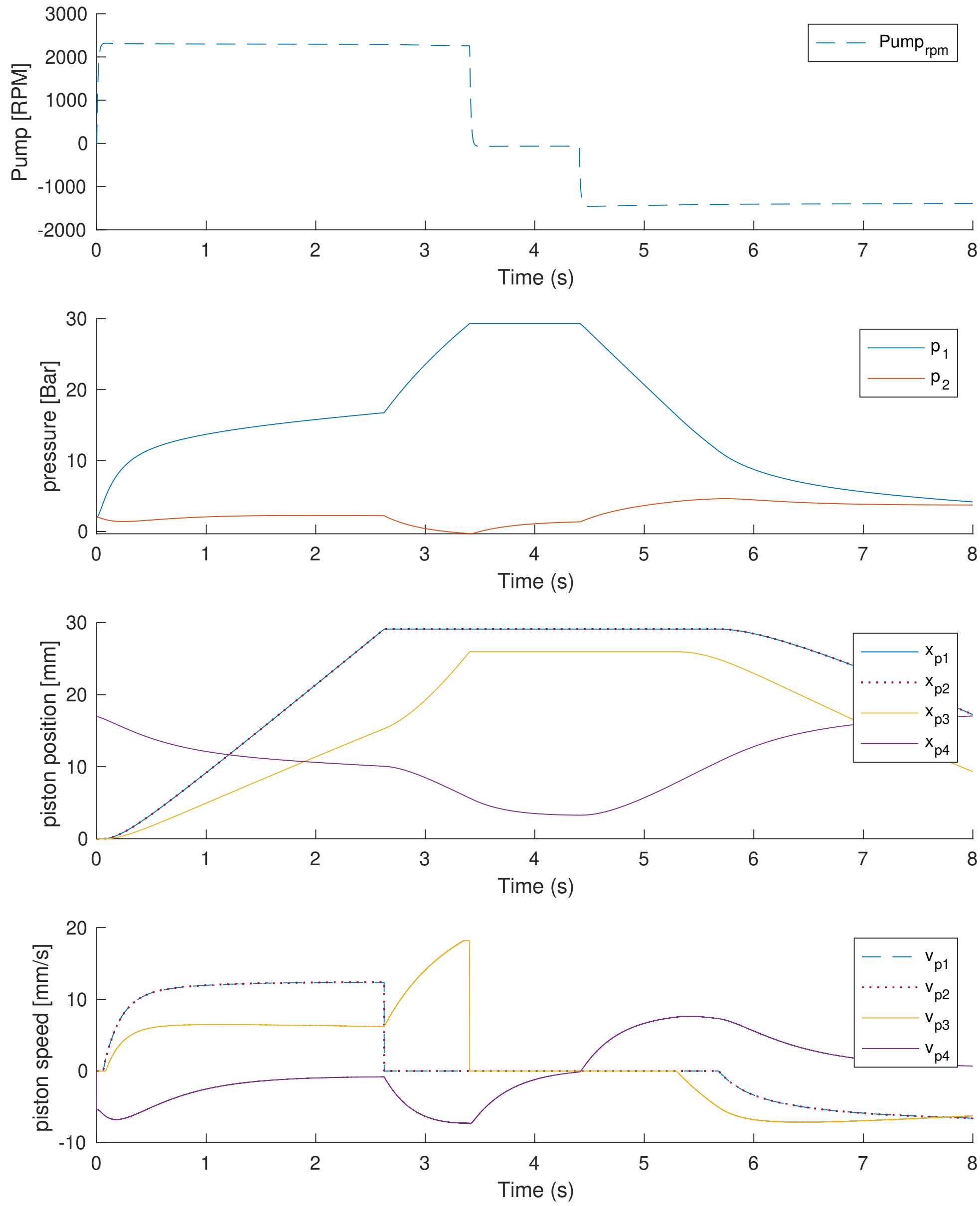

Figure 5.4: Plot of simulation over 8 seconds, with $\beta = 10Mpa$, and closing and opening of the hand. The plot shows the changes in pump rpm, pressure, piston position and piston speed.

Chapter 6

Discussion

The results from the simulation shows that the movements of the piston and pressure dynamics of the simplified model behaves in a way in which the character of the dynamics is similar to how one could expect an idealized $MyHand^{TM}$ model to behave. There are, however, no way of knowing how accurate or inaccurate this behavior is at the moment, since no data from the real hand has been examined at this point. There are some behaviours which seems to be too simplified, like the identical movement of the pistons in cylinder one and two. In reality, one would expect that there will be some difference in the motion between these two cylinders due to pressure losses in the pipes.

And while the behavior seems promising, the values and timings for the states are likely far from the reality of the hand due to the lack of accurate parameters and the simplifications of the model. This model needs to be validated against the real hand before anything conclusive about its behavior and accuracy can be said.

However, the method in this report seems to show promise for modeling and simulating pressure and piston movements within the hand. The model can be adjusted for correct parameters and additional components can be added to the make the model less simplified (like adding pressure losses from pipes and valves, static friction (stiction), and more pressure nodes).

The research questions of this project were to find out if it was possible to create a mathematical model which could be used to gain insight in pressure drops and flow changes, and to potentially be used for automatic control, then to simulate this model using MatLab. As it stands, the current model cannot tell us anything about the pressure drops or flows in the hand. The model needs to be developed further to include pressure losses and be validated against real data of the hand before this can be investigated. For automatic control, I think the current model shows promise, since a model for this purpose often does not need to be entirely accurate. However the model needs to be validated before anything can be said about this this as well.

The strength of the approach in this report (compared to simulation using FEA and CFD) is that you have complete control of the model. You can change it easily, and it is easily simulated. However, the weak side of this approach is that it may be harder to generate an accurate model, and it may be hard to include purely geometrical problems into the model. So if the pressure drops phenomena in MyHand is due to some specific geometric properties (for example the exact shapes of the transmission lines), it will be hard to pick that up with a mathematical model unless the model is very complex.

Chapter 7

Conclusion

The study set out to create a mathematical model which could be simulated in MatLab and tell us something about the pressure drops and flows. It was conducted as a preparatory project for further research which could validate and improve the model by looking at real measured pressure data from $MyHand2^{TM}$. The study resulted in a simplified dynamic model of the pressure dynamics and piston movements, with several simulations carried out in Matlab, simulating the closing and opening of the hand.

The results showed that the behavior of the model makes sense in terms of an idealized hydraulic system of $MyHand2^{TM}$. However, a limitation of this study is that it was not possible to validate the model against measurement data, therefore it is not known how much it resembles the behavior of the real system.

This approach of modeling and simulation shows promise, but it is too early to say if it can give any meaningful insight into how different parameters affects pressure drops and flows of $MyHand2^{TM}$.

7.1 Future work

Further work should focus on validating the simulation results against measurements from the real system. In this context, the assumptions and simplifications made in this report needs to be questioned and the model potentionally revised.

Suggestions for improvements of the model:

- Include pressure loss do to transmission lines and valves
- As a result of pressure loss, the model would need more pressure nodes. A suggestion is to have pne on each side of the pump, one on each side of the cylinders, and one in the equalizer, resulting in 9 total pressure nodes.
- Add stiction to the model for piston movements
- Add realistic input from the motor
- Improve the simulation for the opening of the hand such that all fingers reach the start position

A question that would be worthy to investigate is if the pressure drops depends on specific geometric shapes of the system, and if this can easily be included into the model.

Bibliography

Bakka, B. O., Stray, C. F., Poirters, J. & Olsen, O. (2020), ‘Improved prosthetic functionality through advanced hydraulic design’, *MEC20* .

Bye, P. I. (2013), *Hydraulikk*.

Egeland, O. & Gravdahl, T. (2003), *Modeling and Simulation for Automatic Control*.

Engmark, H. A. (2019), ‘Multifysikk-simulator for hy5-protesen’.

Hy5 (2018*a*), ‘Cad drawing of the negative space in the palm of myhand 2. company confidential.’.

Hy5 (2018*b*), ‘Patent for myhand2’.

Hy5 (2018*c*), ‘Presentation for sopra steria’.

Hy5 (2020), ‘Myhand product details’.
URL: *https://www.hy5.no/myhand*

Love, L., Lind, R. & Jansen, J. (2009), ‘Mesofluidic actuation for articulated finger and hand prosthetics’, *IEEE Internation Conference on Intelligent Robots and Systems* .

Neubauer, B. C. (2017), Principles of Small-Scale Hydraulic Systemsfor Human Assistive Machines, PhD thesis.

Savre (2020), ‘Gear pump’.
URL: *https://savree.com/en/encyclopedia/gear-pump*

Schulz, S., Pylatiuk, C., Reischl, M., Martin, J., Mikut, R. & Bretthauer, G. (2005), ‘A hydraulic driven multifunctional prosthetic hand’, *Robotica* **23**(3).

Wikipedia (2020*a*), ‘Hydraulics’.
URL: *https://en.wikipedia.org/wiki/Hydraulics*

Wikipedia (2020*b*), ‘List of moments of inertia’.
URL: *https://en.wikipedia.org/wiki/List_of_moments_of_inertia*

Xia, J. (2015), Modeling and Analysis of Small-Scale HydraulicSystems, PhD thesis.

Xia, J. & Durfee, W. (2013), ‘Analysis of small-scale hydraulic actuation systems’, *Journal of Mechanical Design* **135**.

Appendix

A simulation.m

```
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Simulation of the hydraulic system in the prosthetic MyHand2 by Hy5.no %
% By Eric Toern                                                          %
% 2020-12-15                                                             %
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear
clc

%% Parameters

% Loads for fingers (1, 2) and thumb (3)
% [N]
p.F_load_1 = 20;
p.F_load_2 = 20;
p.F_load_3 = 20;

% Spring constants in cylinders (1, 2, 3) and equalizer (4)
% [N/mm]
p.k_1 = 1.11 + 0.42;
p.k_2 = 1.11 + 0.42;
p.k_3 = 1.94 + 0.42;
p.k_4 = 4.51;

% Pump & Motor
p.T_m = 2000; % [Nmm] - Motor torque while closing the hand
p.Reverse_Tm = -1200; % [Nmm] - Reverse motor torque while opening the hand
p.D_u = 20; % [mm^3/rad] - Pump displacement
p.m_rod = 30e-3; % [kg] - Mass of motor/pump driving shaft
p.r_rod = 2.5; % [mm] - Radius of motor/pump shaft
p.J_u = 0.5*p.m_rod*p.r_rod^2; % [kg*mm^2] - Moment of intertia of shaft

% Cylinder 1
p.A_p1_1 = 119.7; % [mm^2] - Area of piston in cylinder 1 at side 1
p.A_p1_2 = 154; % [mm^2] - Area of piston in cylinder 1 at side 2
p.m_p1 = 0.020; % [kg] - Mass of piston in cylinder 1
p.V_p1_1_init = 4370.5; % [mm^3] - Volume in cylinder 1 at side 1
p.V_p1_2_init = 10196; % [mm^3] - Volume in cylinder 1 at side 2
p.x_p1_limit = 58.22/2; % [mm] - Length of possible piston movement

% Cylinder 2 (same as cylinder 1)
p.A_p2_1 = p.A_p1_1;
p.A_p2_2 = p.A_p1_2;
p.m_p2 = p.m_p1;
```

```
p.V_p2_1_init = p.V_p1_1_init;
p.V_p2_2_init = p.V_p1_2_init;
p.x_p2_limit = p.x_p1_limit;

% Cylinder 3
p.A_p3_1 = 78.9; % [mm^2]
p.A_p3_2 = 113.1; % [mm^2]
p.m_p3 = 0.020; % [kg]
p.V_p3_1_init = 2703.4; % [mm^3]
p.V_p3_2_init = 6308.7; % [mm^3]
p.x_p3_limit = 51.94/2; % [mm]

% Equalizer
p.A_p4_1 = 289.5; % [mm^2]
p.A_p4_2 = 255; % [mm^2]
p.m_p4 = 0.020; % [kg]
p.V_p4_1_init = 30468; % [mm^3]
p.p_atm = 0.1013; % MPa - atmospheric pressure - [kg/(mm*s^2)]/[N/mm^2]
p.x_p4_limit = 110.2/2; % [mm]

% Oil
p.B_fr = 8.21; % Viscous friction coefficient
p.Beta = 1000; % [MPa] - Bulk Modulus (1000-1500 MPa is normal)

% Events: Detect end of cylinder and holding the grip
p.grip_hold_time = 1; % [s] - How long to hold the closed hand before
    opening
p.hand_has_closed = false; % Turns to true if the all fingers have
    closed
p.solenoid_valve_is_closed = false; % If true, the model simulates
    closed valves (stable pressure in cylinders)
p.event_tol = 1; % [mm] - tolerance for recognizing that the pistons
    have reached the end or start of the cylinders

%% Initial values

% Pump [rad/s]
w_u_init = 0;

% Pressures [MPa]
p1_init = 0.2;
p2_init = 0.2;

% Piston position [mm]
x_p1_init = 0;
x_p2_init = 0;
x_p3_init = 0;
x_p4_init = 17;

% Piston speed [mm/s]
v_p1_init = 0;
v_p2_init = 0;
v_p3_init = 0;
v_p4_init = 0;

% Initial state vector
init_state = [w_u_init, p1_init, p2_init, x_p1_init, x_p2_init,
    x_p3_init, x_p4_init, v_p1_init, v_p2_init, v_p3_init, v_p4_init];
```

```matlab

%% Dynamics
dyn = @(t,x) Dynamics(t,x,p); % see Dynamics.m for parameters and model

%% Events
% The event fires when the piston position (x_p) reaches the end or
% beginning of the cylinders
tol = 2e-10; % Sometimes the event triggered a tiny bit short of the actual end/start
events = @(t,x) Events(x,p); % Events function (see Events.m)

%% Simulation
% Add event function to the ode45 function
options = odeset('Events', events);

% Time (s) to start
tstart = 0;

% Time (s) to end
tfinal = 8;

% Initialize time and state vectors
t = tstart;
x = init_state;

while t(end) < tfinal
    % The simulation stops each time it reaches an event (end of cylinders)
    % When this happens, the model changes by setting dv and dx to zero in
    % the dynamics (see dynamics.m)
    % For the next simulation, the initial value for piston speeds (v) are
    % set to zero. Since the model in the dynamics only gives us the
    % derivatives, this restarting of the simulation with new initial values is
    % necessary for changing the speed state (v) to zero. Otherwise the
    % state would just continue with the value it had before the derivatives
    % were turned to zero.
    % Then the new simulation is ran until a new event triggers (or runs out of time).

    % Start simulation
    [tsim,xsim,te,xe,ie] = ode45(dyn, [t(end),tfinal], x(end, :), options);

    % Append trajectory from latest simulation
    t = cat(1, t, tsim(2:end));
    x = cat(1, x, xsim(2:end, :));

    % Check for event, and set the initial states for the next simulation
    % with the modified model/dynamics
    if x(end,4) >= (p.x_p1_limit - p.event_tol) || x(end,5) <= (0 + p.event_tol) % Piston in cylinder 1 has reached the end or start
        x(end,8) = 0; % Change the speed of the piston to zero
    end
```

```
    if x(end,5) >= (p.x_p2_limit - p.event_tol) || x(end,5) <= (0 + p.event_tol) % Piston in cylinder 2 has reached the end or start
        x(end,9) = 0; % Change the speed of the piston to zero
    end
    if x(end,6) >= (p.x_p3_limit - p.event_tol) || x(end,6) <= (0 + p.event_tol) % Piston in cylinder 3 has reached the end or start
        x(end,10) = 0; % Change the speed of the piston to zero
    end
    if x(end,7) >= (p.x_p4_limit - p.event_tol) || x(end,7) <= (0 + p.event_tol) % Piston in equalizer has reached the end or start
        x(end,11) = 0; % Change the speed of the piston to zero
    end

    % Check if all the fingers has closed, then register and store the time.
    % The model uses these variables to change the input to the pump and pressure dynamics
    if p.hand_has_closed == false && x(end,4) >= (p.x_p1_limit - p.event_tol) && x(end,5) >= (p.x_p2_limit - p.event_tol) && x(end,6) >= (p.x_p3_limit - p.event_tol)
        p.hand_has_closed = true; % register that the hand has closed
        p.time_when_hand_closed = t(end); % store the time that the hand closed
        dyn = @(t,x) Dynamics(t,x,p); % add new parameters to the dynamics
    end

    % Print the time when the event triggered
    fprintf("Time of event detection: %f seconds\n", t(end));
end

%% Plot
close all

figure('WindowState', 'maximized');

% Pump rotation
subplot(4,1,1); hold on
w_rpm = x(:,1)*60/(2*pi); % Convert to rpm
plot(t,w_rpm,'--')
xlabel('Time (s)');
ylabel('Pump [RPM]');
legend('Pump_{rpm}')

% Pressures
subplot(4,1,2); hold on
p1_bar = x(:,2)*10; % Convert from MPa to Bar
p2_bar = x(:,3)*10; % Convert from MPa to Bar
plot(t,p1_bar,'-',t,p2_bar,'-')
xlabel('Time (s)');
ylabel('pressure [Bar]');
legend('p_1','p_2')

% Piston position
subplot(4,1,3); hold on
plot(t,x(:,4),'-', 'Color', '#0072BD')
plot(t,x(:,5),':', 'LineWidth', 1, 'Color', '#A2142F')
plot(t,x(:,6),'-', 'Color', '#EDB120')
```

```
plot(t,x(:,7),'-', 'Color', '#7E2F8E')
xlabel('Time (s)');
ylabel('piston position [mm]');
legend('x_{p1}', 'x_{p2}', 'x_{p3}', 'x_{p4}')

% Piston speed
subplot(4,1,4); hold on
plot(t,x(:,8),'--', 'Color', '#0072BD')
plot(t,x(:,9),':', 'LineWidth', 1, 'Color', '#A2142F')
plot(t,x(:,10),'-', 'Color', '#EDB120')
plot(t,x(:,11),'-', 'Color', '#7E2F8E')
xlabel('Time (s)');
ylabel('piston speed [mm/s]');
legend('v_{p1}', 'v_{p2}', 'v_{p3}', 'v_{p4}')
```

B Dynamics.m

```
function [ dstate ] = Dynamics(t,state,p)
    %% Clarifying state variables
    w_u = state(1); % [rad/s]
    p_1 = state(2); % [MPa]
    p_2 = state(3); % [MPa]
    x_p1 = state(4); % [mm]
    x_p2 = state(5); % [mm]
    x_p3 = state(6); % [mm]
    x_p4 = state(7); % [mm]
    v_p1 = state(8); % [mm/s]
    v_p2 = state(9); % [mm/s]
    v_p3 = state(10); % [mm/s]
    v_p4 = state(11); % [mm/s]


    %% Dynamics

    % Pump dynamics
    %%%%%%%%%%%%%%%%%

    % Flow from pump
    q_1 = p.D_u*w_u; % [mm^3/s]

    % Check if hand has closed during the simulation, then set the
        motor torque
    if p.hand_has_closed
        % if the simulation has not yet reached the end of the gripping
            time:
        if (t-p.time_when_hand_closed) <= p.grip_hold_time
            p.T_m = 0; % Set motor torque to zero
            p.solenoid_valve_is_closed = true; % Close the valve
        % if the thumb has opened completely (reached the start of the
            cylinder):
        elseif x_p3 <= (0 + p.event_tol)
            p.T_m = 0; % Set motor torque to zero
        % if the hand is in the opening phase
        elseif (t-p.time_when_hand_closed) > p.grip_hold_time
            p.T_m = p.Reverse_Tm; % Reverse pump to open the hand
            p.solenoid_valve_is_closed = false; % Open the valve
        end
    end
```

```matlab

    % Calculate derivative of the pumps rotational speed
    dw_u = (-p.B_fr*w_u - p.D_u*(p_1-p_2) + p.T_m) / p.J_u; % [rad/s^2]

    % Pressure dynamics
    %%%%%%%%%%%%%%%%%%%%%%

    % Volumes at current time step
    V_tot_1 = p.V_p1_1_init + p.V_p2_1_init + p.V_p3_1_init + p.A_p1_1*x_p1 + p.A_p2_1*x_p2 + p.A_p3_1*x_p3; % [mm^3]
    V_tot_2 = p.V_p1_2_init + p.V_p2_2_init + p.V_p3_2_init + p.V_p4_1_init - p.A_p1_2*x_p1 - p.A_p2_2*x_p2 - p.A_p3_2*x_p3 + p.A_p4_1*x_p4; % [mm^3]

    % Flows and volume changes at current time step
    q_tot_1 = q_1 - p.A_p1_2*v_p1 - p.A_p2_2*v_p2 - p.A_p3_2*v_p3; % [mm^3/s]
    q_tot_2 = -q_1 + p.A_p1_2*v_p1 + p.A_p2_2*v_p2 + p.A_p3_2*v_p3 - p.A_p4_1*v_p4; % [mm^3/s]

    % Calculate the derivate of pressure 1 and 2
    dp_1 = p.Beta*q_tot_1/V_tot_1; % [MPa/s]
    dp_2 = p.Beta*q_tot_2/V_tot_2; % % [MPa/s]

    % Simulate the solenoid valve when hand is closed and pump has stopped,
    % unabling the flow to reverse (and thus the pressure to stay constant)
    if p.solenoid_valve_is_closed == true
        dp_1 = 0;
    end

    % Motion dynamics for the pistons
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    % Calculate sum of forces on piston 1, 2, 3, 4
    F_sum_1 = p.A_p1_1*p_1 - p.A_p1_2*p_2 - p.F_load_1 - p.k_1*x_p1; % [N]
    F_sum_2 = p.A_p2_1*p_1 - p.A_p2_2*p_2 - p.F_load_2 - p.k_2*x_p2; % [N]
    F_sum_3 = p.A_p3_1*p_1 - p.A_p3_2*p_2 - p.F_load_3 - p.k_3*x_p3; % [N]
    F_sum_4 = p.A_p4_1*p_2 - p.A_p4_2*p.p_atm - p.k_4*x_p4; % [N]

    % Check if the piston have reached the end or start of the
    % cylinder. If so, the model needs to change by adding an opposite
    % reacting force, halting the piston:

    % Piston in cylinder 1
    if x_p1 >= (p.x_p1_limit - p.event_tol) && F_sum_1 > 0 && v_p1 >= 0 % If pushing against the end of cylinder
        dv_p1 = 0; % dv_p1 = F_sum_1 - F_opposite reaction = 0
    elseif x_p1 <= (0 + p.event_tol) && F_sum_1 <= 0 && v_p1 < 0 % If pushing against the start of cylinder
        dv_p1 = 0; % dv_p1 = F_sum_1 - F_opposite reaction = 0
    else
        dv_p1 = (p.A_p1_1*p_1 - p.A_p1_2*p_2 - p.B_fr*v_p1 - p.F_load_1 - p.k_1*x_p1)/p.m_p1; % [m/s^2] (converted at bottom)
```

```
    end

    % Piston in cylinder 2
    if x_p2 >= (p.x_p2_limit - p.event_tol) && F_sum_2 >= 0 && v_p2 >= 0
        dv_p2 = 0;
    elseif x_p2 <= (0 + p.event_tol) && F_sum_2 <= 0 && v_p2 < 0
        dv_p2 = 0;
    else
        dv_p2 = (p.A_p2_1*p_1 - p.A_p2_2*p_2 - p.B_fr*v_p2 - p.F_load_2 - p.k_2*x_p2)/p.m_p2; % [m/s^2] (converted at bottom)
    end

    % Piston in cylinder 3
    if x_p3 >= (p.x_p3_limit - p.event_tol) && F_sum_3 >= 0 && v_p3 >= 0
        dv_p3 = 0;
    elseif x_p3 <= (0 + p.event_tol) && F_sum_3 <= 0 && v_p3 < 0
        dv_p3 = 0;
    else
        dv_p3 = (p.A_p3_1*p_1 - p.A_p3_2*p_2 - p.B_fr*v_p3 - p.F_load_3 - p.k_3*x_p3)/p.m_p3; % [m/s^2] (converted at bottom)
    end

    % Piston in cylinder 4
    if x_p4 >= (p.x_p4_limit - p.event_tol) && F_sum_4 >= 0 && v_p4 >= 0
        dv_p4 = 0;
    elseif x_p4 <= (0 + p.event_tol) && F_sum_4 <= 0 && v_p4 < 0
        dv_p4 = 0;
    else
        dv_p4 = (p.A_p4_1*p_2 - p.A_p4_2*p.p_atm - p.B_fr*v_p4 - p.k_4*x_p4)/p.m_p4; % [m/s^2] (converted at bottom)
    end

    dx_p1 = v_p1; % [mm/s]
    dx_p2 = v_p2; % [mm/s]
    dx_p3 = v_p3; % [mm/s]
    dx_p4 = v_p4; % [mm/s]

    % Convert units and return the state derivatives
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    % Converting the acceleration from m/s^2 to mm/s^2
    dv_p1 = dv_p1 * 1000; % [mm/s^2]
    dv_p2 = dv_p2 * 1000; % [mm/s^2]
    dv_p3 = dv_p3 * 1000; % [mm/s^2]
    dv_p4 = dv_p4 * 1000; % [mm/s^2]

    dstate = [dw_u, dp_1, dp_2, dx_p1, dx_p2, dx_p3, dx_p4, dv_p1, dv_p2, dv_p3, dv_p4]';
end
```

C Events.m

```
function [value,isterminal,direction] = Events(state, p)

x_p1 = state(4);
```

```
x_p2 = state(5);
x_p3 = state(6);
x_p4 = state(7);

% Detect event when one of the vector values in bector 'value' becomes
    zero
value = [x_p1 - p.x_p1_limit, x_p2 - p.x_p2_limit, x_p3 - p.x_p3_limit,
     x_p4 - p.x_p4_limit, x_p1, x_p2, x_p3, x_p4];

% Stop the integration when the event is detected
isterminal = [1, 1, 1, 1, 1, 1, 1, 1];

% Direction for which the event is valid
direction = [1, 1, 1, 1, -1, -1, -1, -1];
```